# STOCHASTIC ANALYSIS OF GROUND RESPONSE VARIABILITY FOR SEISMIC DESIGN OF BURIED LIFELINE STRUCTURES

by

Takanori Harada* , Masanobu Shinozuka**

ABSTRACT

The ground response variabilities in space and time, especially in space, have been analysed using a stochastic ground surface model. The intent is to quantify the spatial variation of ground motions for seismic analysis and design of buried lifeline structures such as gas or water supply pipeline. The ground is assumed to be infinitely horizontal soil layers with stochastic soil thicknesses and soil properties which are supported on rigid base rock. The earthquake motion is subjected to the ground from rigid base rock with time lag x/c where x = horizontal coordinate and c = wave speed in x-direction. From this study, it is found that the spatial correlation function of response displacement at ground surface is a function of response velocity spectrum, spatial correlation function of predominant ground frequency, mean value of equivalent damping ratio of ground, mean value of predominant ground frequency, and wave speed in horizontal direction.

## 1. INTRODUCTION

In contrast to the earthquake-resistant designs of above-ground structures where the inertial forces induced by ground acceleration are the main consideration, the spatial variation of the ground motion is of primary importance for buried lifeline structures such as pipelines and tunnels. Consistent with this observation, the response-displacement method was devised ( Kubo, et al.,1979 ) and is widely used for the earthquake-resistant design of underground structures in Japan. However, the state-of-the-art of quantifying the spatial variability of earthquake ground motion and the permanent ground deformation resulting therefrom still leaves much to be desired.

It appears particularly important, now that dense array seismic data has been beginning to become available, to analyze the spatial variation of earthquake ground motions. In this context, several empirical studies have been available from a direct statistical point of view ( Loh, et al.,1982, Vanmarcke and Harichandran, 1984, Harada, 1984, Harada and Shinozuka, 1986 ). The permanent ground deformation resulting from earthquakes has also been collected and analysed from statistical point of view ( Harada and Iwasaki et al.,1985 ). On the other hand, it is desirable to develop mechanistic model to study the ground motion problem from an analytical standpoint. Such models could be used not only to interpret seismic array data in terms of wave content but also as an aid in the development of site-specific

* Associate Professor, Dept. of Civil Engineering, Miyazaki University, Miyazaki 889-21, JAPAN.
** Renwick Professor of Civil Engineering, Dept. of Civil Engineering and Engineering Mechanics, Columbia University, New York, N.Y. 10027, U.S.A.

earthquake resistant design parameters. From this point of view, the spatial coherence of earthquake ground motions has been studied using a stochastic model of SH waves travelling in a homogeneous soil with varying angle ( Kausel and Paris,1984 ). The differential ground motions have been studied using the sophisticated physical models of earthquake faulting (Bard and Bouchon, 1980, Zerva and Ang et al.,1985). In this paper, the ground response variabilities in space and time, especially in space, have been analysed using a stochastic ground surface model.

## 2. HOMOGENEOUS STOCHASTIC HORIZONTAL GROUND

In a conventional ground response analysis, it is common to model the ground as the layers with constant thicknesses and soil properties. However, in a real profile schematically shown in Fig.1 where j denotes a typical soil layer and x,z measure horizontal and vertical space coordinates, the soil thickness Hj(x) of the j-th layer may vary randomly from point to point as a function of x. Similarly the representative soil property q(x,z) may be a random function of x and z. In considering a nominally or almost homogeneous horizontal soil layers as depicted in Fig.1, it may be agreeable, at least for first approximation, to assume that the soil thickness Hj(x) and the soil property q(x,z) are random functions of only horizontal x-axis such as

$$H_j(x) = H_j[1 + f_{Hj}(x)], \quad q(x, z) = q(z)[1 + f_q(x)] \tag{1}$$

where Hj and q(z) are the expected value (mean value) of Hj(x) and q(x,z) with respect to x, respectively, that is

$$E[H_j(x)] = H_j, \quad E[q(x, z)] = q(z) \tag{2}$$

Eventually, $f_{Hj}(x)$ and fq(x) in Eq.1 represent the stochastic fluctuation of Hj(x) and q(x,z) along horizontal x-axis with mean zero, $E[f_{Hj}(x)] = 0$ and E[fq(x)]=0. The term "nominally or almost homogeneous horizontal soil layers" may mean $|f_{Hj}(x)| \ll 1, |fj(x)| \ll 1$, in Eq.1.

## 3. EQUATIONS OF MOTION FOR HOMOGENEOUS STOCHASTIC HORIZONTAL GROUND

Consider the homogeneous stochastic horizontal ground supported by rigid bed rock and subjected to earthquake ground motion as shown in Fig.2. The total soil depth is assumed constant H. The input earthquake ground motion is assumed to be a stationary random wave propagating with speed c in x-direction being represented by

$$u_b(x, t) = u_b(t - x/c) \tag{3}$$

The relative displacement at location of x and z with respect to the input motion $u_b(x,t)$ at bed rock is denoted by $u^r(x,z,t)$. Then, the total response displacement u(x,z,t) can be expressed as

$$u(x,z,t) = u_b(t - x/c) + u^r(x,z,t) \tag{4}$$

Employing the generalized displacement concept, Eq.4 can be written as,

$$u(x,z,t) = u_b(t - x/c) + u^*(x,t)\ \psi(z) \tag{5}$$

where

$$u^r(x,z,t) = u^*(x,t)\ \psi(z), \quad \psi(0) = 1 \tag{6}$$

In Eqs.5 and 6, u*(x,t) is the amplitude of motion or the generalized displacement, and $\psi(z)$ is the given shape function or the assumed mode function. It is usually convenient, although not essential, to normalize the shape function as given in Eq.6. The shape function must satisfy the geometric boundary conditions. In the ground considering at hand, the geometric boundary condition is given such as (see Fig.2)

$$u^r(x,z,t) = 0 \quad \text{at } z = H \tag{7}$$

The shape function can be arbitrarily assumed, provided it satisfies the geometric boundary condition. However, it is better to assume a shape that may be expected to be similar to the deformation of the ground subjected to earthquake motion at bed rock. For simplicity, in this paper, the shape function is assumed as

$$\psi(z) = \cos[\pi z/(2H)] \tag{8}$$

This shape function in Eq.8 corresponds to the first mode shape of a single homogeneous infinite horizontal layer lying on rigid bed rock. Of course, Eq.8 satisfies the normalization and the geometric boundary condition given by Eqs.6 and 7.

Under the conditions described above, consider the forces acting on a small soil element (dx*dz) within the ground as shown in Fig.2. The inertial force $F_I$ and the restoring force $F_R$ are given by definitions as

$$F_I = \rho(x,z)\,\ddot{u}(x,z,t)\,dxdz\ ,\quad F_R = k(x,z)\,u^r(x,z,t)\,dxdz \tag{9}$$

where $\ddot{u}(x,z,t) = \partial^2 u(x,z,t)/\partial t^2$ = the absolute acceleration of soil particle at location x and z, $\rho(x,z)$= the soil mass per unit area at x and z , and $k(x,z)$= the soil resistance per unit area at x and z. Recalling the definition of the homogeneous stochastic horizontal ground given in Eqs.1 and 2, the soil mass and resistance can be expressed by

$$\rho(x,z) = \rho_z(z)[1 + f_\rho(x)],\quad k(x,z) = k_z(z)[1 + f_k(x)] \tag{10}$$

where $\rho_z(z)$ and $k_z(z)$ represent the means of $\rho(x,z)$ and $k(x,z)$ being the deterministic function of z. The functions $f_\rho(x)$ and $f_k(x)$ are the random function of space coordinate x with zero means.

By utilizing the concept that the virtual work $\delta W$ done by the virtual displacement $\delta u^r$ is zero, that is,

$$\delta W = \int_0^H (F_I + F_R)\,\delta u^r = 0 \tag{11}$$

where $\delta u^r = \psi(z)\delta u^*(x,t)$ since $\psi(z)$ is the given shape function, and substituting Eqs.5 to 10 into Eq.11, and introducing the equivalent damping ratio $h^*(x)$ to take into account for the vibration energy loss due to wave propagation as well as the hysteresis behavior of soil stress-strain curve under dynamic loadings, one can obtain

$$\ddot{u}^*(x,t) + 2h^*(x)\omega^*(x)\dot{u}^*(x,t) + [\omega^*(x)]^2 u^*(x,t) = -\beta\ddot{u}_b(t - x/c) \tag{12}$$

where $\omega^*(x)$ = the ground natural circular frequency (rad/s), $\beta$ = the participation factor. They are given such as

$$\omega^*(x) = \sqrt{\frac{[1 + f_k(x)]\int_0^H k_z(z)\psi^2(z)dz}{[1 + f_\rho(x)]\int_0^H \rho_z(z)\psi^2(z)dz}} \qquad \beta = \frac{\int_0^H \rho_z(z)\psi(z)dz}{\int_0^H \rho_z(z)\psi^2(z)dz} \tag{13}$$

Alternatively, for formal purpose, the ground predominant (natural) frequency $\omega^*(x)$ and the equivalent damping ratio $h^*(x)$ may be also expressed similar to Eq.1 such as

$$\omega^*(x) = \omega_0[1 + f(x)], \quad h^*(x) = h_0[1 + h(x)] \tag{14}$$

where $\omega_0$ and $h_0$ are the means of $\omega^*(x)$ and $h^*(x)$, and $f(x)$ and $h(x)$ are the homogeneous stochastic processes with zero means. For the expression as given in Eq.14, it is easy to show that

$$\delta_{\omega^*} = \sigma_{ff} , \quad \delta_{h^*} = \sigma_{hh} \tag{15}$$

where $\delta_{\omega^*}$ and $\delta_{h^*}$ are the coefficients of variation of $\omega^*(x)$ and $h^*(x)$, respectively. And also $\sigma_{ff}$ and $\sigma_{hh}$ are the standard deviations of $f(x)$ and $h(x)$, respectively.

## 4.APPLICATION OF SOME RESULTS

Equation 12 is a nonlinear equation of space coordinate x, while a linear equation of time t. Hence, using a linearization of Eq.12 with respect to x, one obtains approximately the two important functions such as (Harada and Shinozuka, 1986),

$$P_{uu}(\xi,\omega) = S_{u_b u_b}(\omega) e^{-i\omega\xi/c} [\{\omega_0^4 + (2\beta + 4h_0^2 - 2)\omega_0^2\omega^2 + (\beta - 1)^2\omega^4\} |H(\omega)|^2 + 4\beta^2\omega_0^4\omega^4 R_{ff}(\xi) |H(\omega)|^4] \tag{16}$$

and,

$$R_{uu}(\xi) = R_{u_1u_1}(\xi) + R_{u_2u_2}(\xi) = (\beta^2 + 4h_0^2)\left[\frac{S_V(\omega_0,h_0)}{PFA_t \cdot \omega_0}\right]^2 e^{-\left(\frac{\omega_0 \xi}{\sqrt{2}\, c}\right)^2} + 4\beta^2(1 + 4h_0^2) R_{ff}(\xi) \times \left[\frac{S_V(\omega_0, 8h_0^3)}{PFA_t \cdot \omega_0}\right]^2 e^{-\left(\frac{\omega_0 \xi}{\sqrt{2}\, c}\right)^2} \tag{17}$$

where $P_{uu}(\xi,\omega)$ = the temporal spectral density spatial correlation function of $u(x,t)$, $R_{uu}(\xi)$ = the spatial correlation function of $u(x,t)$, $H(\omega)$ = the transfer function defined as $1/(\omega_0^2 - \omega^2 + i2h_0\omega_0\omega)$, $S_{u_bu_b}(\omega)$ = the power spectral density function of input displacement $u_b(t)$, $R_{ff}(\xi)$ = the spatial correlation function of $f(x)$ (see Eq.14), $S_V(a_1,a_2)$ = the response spectrum value at $a_1,a_2$, and $PFA_t$ = the peak factor = $PFA_t^* + 0.5772/PFA_t^*$ with

$$PFA_t^* = \sqrt{2 \ln(\omega_0 T / \Pi)} , \quad T = \text{duration of } u_b(t) \tag{18}$$

The following example may demonstrate one of the applications of Eq.17 for estimating a site specific spatial variation of ground motion.

Consider a site where soil conditions are shown in Fig.3 and Table 1. The bottom of the ground is assumed to be rigid. The predominant ground frequency $\omega^*(x_n)$ (n=1,2,..,60) may be estimated by (an extention of Okamoto's equation, 1984),

$$\hat{\omega}^*(x_n) = 2\Pi \hat{f}^*(x_n) = 2\Pi/\hat{T}^*(x_n) = 2\Pi/(4 \sum_{j=1}^{n} H_j(x_n)/v_{sj}(x_n)) \tag{19}$$

The mean value and the coefficient of variation of $\omega^*(x_n)$ are estimated as $\hat{\omega}_0 = 2\Pi\hat{f}_0 = 5.59$ (rad/s), ($T_0 = 1.12$ s), $\hat{\delta}_{\omega^*} = 0.085$. The sample spatial correlation function $\hat{R}_{ff}(\xi_k)$ of f(x) is calculated by

$$\hat{R}_{ff}(\xi_k) = \frac{1}{N - k} \sum_{n=1}^{N-k} [\hat{\omega}^*(x_n + \xi_k) - \hat{\omega}_0][\hat{\omega}^*(x_n) - \hat{\omega}_0] \qquad (20)$$

where N= the total number of soil sections (60). The resulting spatial correlation coefficient $\hat{R}_{ff}(\xi_k)/\hat{R}_{ff}(0)$ ( $\hat{R}_{ff}(0) = \hat{\sigma}_{ff}^2 = \hat{\delta}_{\omega^*}^2 = 0.085 \times 0.085$, see Eq.15), is plotted by solid curve in Fig.4. Parenthetically, the ground response displacements are computed for the same ground model using a finite element method with 5% of critical damping in each soil layer (Harada and Sakamoto, 1985). From the response displacement u(x,t) at ground surface, the spatial correlation function can be esimated, and the resulting spatial correlation coefficient is also plotted in Fig.4 by dashed curve. The relatively nice agreement between the spatial correlation coefficient calculated by FEM and that estimated from $\hat{\omega}^*(x_n)$ indicates a validity of Eq.17 since in Eq.17 the spatial correlation function of u(x,t) is proportional to the spatial correlation function $R_{ff}(\xi)$ of the predominant ground frequency for c = infinity as in the case of FEM response analysis. To approximately represent the behavior of spatial correlation function $\hat{R}_{ff}(\xi_k)$ indicated in Fig.4 (solid curve), an appropriate analytical form is assumed such as

$$R_{ff}(\xi) = \sigma_{ff}^2 [1 - 2(\xi/b)^2] e^{-(\xi/b)^2} \qquad (21)$$

with $\sigma_{ff} = \delta_{\omega^*} = 0.085$, b = 141.42 (m). The value of b is determined in such a way that Eq. 21 takes zero at $\xi$ = 100 m.

Substituting Eq.21 into Eq.17, the spatial correlation function $R_{uu}(\xi)$ of ground surface displacement u(x,t) for the site with soil conditions as shown in Fig.3 can be obtained. From the spatial correlation function, one can obtain the ground strain and the relative displacement between two points on ground surface (ground deformation spectrum, Harada and Shinozuka, 1986) as a function of relative distance and site specific parameters such as earthquake intensity (response velocity spectrum), seismic wave speed in horizontal direction, and spatial variability of ground predominant frequency (spatial correlation function of ground frequency). More details and applications can be seen in Ref.(9). Finally, the authors acknowledge the support provided by the Tokyo Gas Company, Ltd., Tokyo, Japan. Particular thanks are due to Mr. N. Nishio, Manager and Chief Researcher, Tokyo Gas Company, for his valuable comments on the technical contents of this paper. Also, this work was supported by the National Science Foundation under Grant No. CEE-84-14205.

REFERENCES

(1) Kubo, K., Katayama, T. and M. Ohashi, "Lifeline Earthquake Engineering in Japan," Journal of the Technical Councils, ASCE, Vol. 105 (TC1), 221-238, 1979.

(2) Loh, C.H., Penzien, J. and Y.B. Tsai, "Engineering Analyses of SMART-1 Array Accelerograms," Earthquake Engineering and Structural Dynamics, Vol. 10, 575-591, 1982.

(3) Vanmarcke, E.H. and R.S. Harichandran, "Models of The Spatial Variation of Ground Motion for Seismic Analysis of Structures," Proceedings of the 8th World Conference on Earthquake Engineering, 597-604, 1984.

(4) Harada, T., "Probabilistic Modeling of Spatial Variation of Strong Earthquake Ground Displacements," Proceedings of the 8th World Conference on Earthquake Engineering, 605-612, 1984.

(5) Harada, T. and M. Shinozuka, "Ground Deformation Spectra," Proceedings of the 3rd U.S. National Conference on Earthquake Engineering, 1986.

(6) Harada, T. and T. Iwasaki, et al., "Preliminary Study on Spatial Variation of Ground Deformation for Seismic Design of Buried Lifeline Structures," Public Works Research Institute Report, No. 2143, Ministry of Construction, 1984 (in Japanese).

(7) Kausel, E. and A. Pais, "Deconvolution of Stochastic SH-Wave Motions in Soil Deposits," Research Report R84-09, Dept. of Civil Engineering, Massachusetts Institute of Technology, 1984.

(8) Zerva, A., Ang, A.H-S. and Y.K. Wen, "A Study of Seismic Ground Motion for Lifeline Response Analysis," Structural research Series No. 521, Dept. of Civil Engineering, University of Illinois at Urbana-Champaign, 1985.

(9) Harada, T. and M. Shinozuka, "Stochastic Analysis of Ground Response Variation for Seismic Design and Analysis of Buried Lifeline Structures," (To be in the Technical Report of Columbia University, 1986).

(10) Harada, T. and K. Sakamoto, "A Computer Program SPATIAL-V for Seismic Analysis of Structures," Bulletin of the Faculty of Engineering, Miyazaki University, No. 31, 1985.


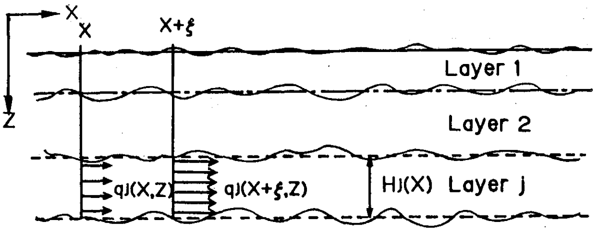


Fig.1 Stochastic Ground

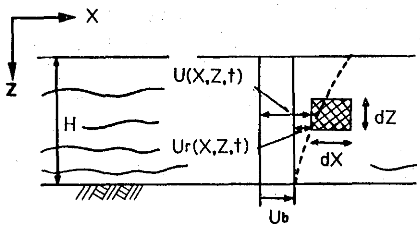


Fig. 2 Model Ground

Fig. 3 Ground Used in Example

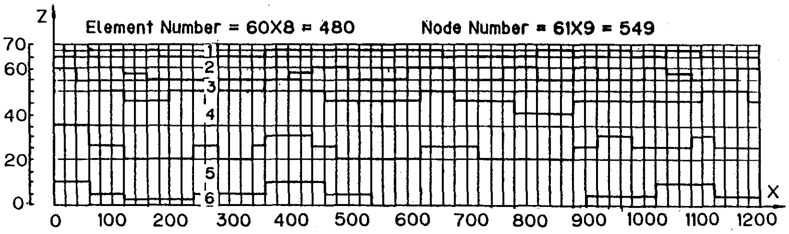


Table 1 Soil Properties Used in Example

| Layer (1) | Soil Mass g/cm$^2$ (2) | Poisson Ratio (3) | Shear Modulus kg/cm$^2$ (4) | Shear Wave Velocity m/s (5) |
|---|---|---|---|---|
| 1 Sand | 1.80 | 0.48 | 133.0 | 85 |
| 2 Sand | 1.70 | 0.48 | 287.0 | 125 |
| 3 Clay | 1.50 | 0.48 | 612.0 | 200 |
| 4 Gravel | 1.90 | 0.48 | 2050.0 | 325 |
| 5 Sand-stone | 2.10 | 0.48 | 5360.0 | 500 |
| 6 Sand-stone | 2.20 | 0.48 | 14367.0 | 800 |

Fig. 4 Spatial Correlation Functions