Transmittance Properties in Multilayered Periodic Structures

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Abstract

Transmittance properties from a multilayered periodic structures is presented using Moment Method together with Lattice Sums and Transfer Matrix Method (TMM). The arrangement constitutes a two-dimensional (2-D) photonic band gap (PBG) structure. The optical behavior of multilayered periodic array of dielectric cylinders are characterized by it reflectanc and the effect of the composite material between the background with various array patterns are shown. Multiple stop bands formation that depicted Distributed Bragg Reflecto (DBR) is achieved for filter application. The effect of electromagnetic microcavity is shown when two or three DBRs are cascaded .

Key Words :
Multilayered Periodic Structures, Moment Method, Lattice Sum's Technique, Transfer Matrix Method, Distributed Bragg Reflecto

1. Introduction

The analysis of a multilayered periodic structure composed of cylindrical dielectric objects is presented in this paper. Such structures have a stop band in their transmission characteristics and therefore called electromagnetic band gap (EBG) structure in the microwave range or photonic band gap (PBG) structures in the infrared wavelength range. Recently, EBGs and PBGs are of great interest due to their extraordinary properties and potential application and several research efforts have been undertaken to investigate the scattering properties from such array structures [1)]. Periodic array structures have many applications in the design of antennas and waveguide for bandwidth enhancement or dual-band applications [2)]. When treating composite material for the case wherein the periodic array cylinders are in background medium of different material, the material composition possess resonance anomalies and unique frequency-selectivity [3),4)] base on mode analysis method.

A structure with multiple layers containing one or more periodic arrays in separate layers can be used to obtain high-efficiency reflectio or transmission filter depending on whether the thicknesses and dielectric constants of the composite materials are chosen to yield high transmission outside the stopband (i.e., anti-reflectio (AR) design) or low transmission outside the passband (i.e., high-reflectio (HR) design), respectively [5)]. The filte characteristics are tailored by adjusting the parameters of the device.The sidebands can be made arbitrarily low and extended over a large frequency range by adding layers with dielectric constants and thicknesses obeying AR/HR conditions [6),7)].

In this research, we investigated a multilayered periodic structure of dielectric cylinders with two-dimensional PBG. We firs of all analyze a single array by periodic moment method (PMM) [8)] and integral equation formulation [9)]. The dense matrix coefficient constitute the periodic Green's function which is then simplifie using the lattice sums technique [10)]. For a multilayered structure, the scattering properties of the entire 2-D structure are obtained by cascading the 1-D arrays using the Transfer Matrix Method (TMM). In order to cascade the layers together, Fresnel reflectio and transmission matrix are used together with the phase correction between layers [11),12)]. The matrix computation can be stabilized by using matrix decomposition method to eigenvalues and

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eigenvectors that enables us to easily locate the stop band formation. Detailed investigation on the stop band width is carry out for different structural parameters.

2. Methodology and Formulation

Consider a plane wave at an angle θ_i from the upper vertical plane of the firs layer as shown in Fig.1. This geometry may contain any number of layers with periodic array of arbitrary shapes. The cylinders of relative permittivities ε_r are embedded in a medium ε_{rb}. The radius of the cylinder is assumed to be a. The layers alternate itself after every period H in the y-direction and the periodicity of each array is d along the x-axis with the structure uniform in the z direction. In general, we assume a position shift of a distance d_1 in the lateral direction so that it is possible to investigate the effect of square and triangular array pattern by adjusting the parameter d_1. For a square pattern $d_1 = 0$ and for a triangular pattern $d_1 = 0.5d$. It is noted that for an arbitrary value d_1, the periodicity is not in the y direction but actually has a value $\sqrt{d_1^2 + H^2}$ along the direction $\sin^{-1} d_1/H$ from the y-axis.

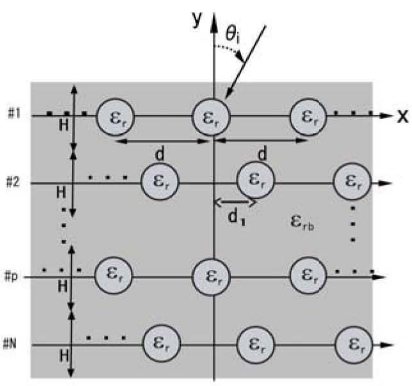

図 1: Geometric configuratio of multilayered periodic structures.

We firs of all mention the Moment Method based on volume integral formulation for the scattering of a periodic array of the arbitrary shaped cylinders situated in a layer medium. For the TM case, the E-fiel is polarized along the $\hat{z}$ direction, assuming the incident plane wave given by

$$\mathbf{E} = \hat{z}\exp(j(k_0 \sin\theta_i x + k_0 \cos\theta_i y)) \tag{1}$$

where k_0 is the wave number in free space, x and y are the position of the observation points within the cylinder. Detailed formulation for the integral equation in free space has been shown in [9]. The same procedure is extended to the integral equation when the periodic scatter is embedded in a background medium as follows:

$$E(\rho) = -j\frac{k_0^2}{4}\sum_{l=-\infty}^{\infty}\int_{s_l} H_0^{(2)}(k_b\rho_l) \times E(\rho_0)\exp(-jk_x ld)(\varepsilon_r - \varepsilon_{rb})ds_l + E_i(\rho) \tag{2}$$

where $H_0^{(2)}(k_b\rho_l)$ is the zeroth-order Hankel function of the second kind, k_b is the wave number in medium and ρ_l is the distance between the source and observation points. ε_r is the relative permittivity of the cylinder array and ε_{rb} is the relative permittivity of the medium where the arrays are situation. The resulting matrix equations given in Eq.3 are formed from the integral equations that can be solved using GMRES [13] as the iterative solver.

$$\sum_{n=1}^{N} C_{mn}E_n = E_m^i \qquad m = 1,\cdots,N \tag{3}$$

The matrix coefficient constitutes the periodic Green's function which can be evaluated using the Lattice Sum's techniques already discussed in [10].

2.1 Transfer Matrix Method

TMM is a most wildly used mathematical study of wave transmission in 1-D structure because it allows the calculation of reflect vity and transmission spectra. In this formulation, the reflectanc and transmittance of plane waves through the multilayered structure are calculated from the 2×2 matrix elements given by:

$$\begin{pmatrix} A_0 \\ B_0 \end{pmatrix} = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}\begin{pmatrix} A_N \\ 0 \end{pmatrix} \tag{4}$$

Therefore;

$$A_0 = M_{11}A_N\ , \quad B_0 = M_{21}A_N \tag{5}$$

The reflectio coefficient R is define as ratio between the backward and forward traveling wave and transmission coefficient T for the multilayered periodic structures are given by:

$$R = \frac{M_{21}}{M_{11}} \equiv \frac{B_0}{A_0}\ , \quad T = \frac{1}{M_{11}} \equiv \frac{A_N}{A_0} \tag{6}$$

3. Numerical Results

The transmittance characteristic of the multilayered periodic structure are calculated. The results are shown for the wavelength range $0.5 < \lambda/\lambda_0 < 2.6$ under normal incidence and the condition $d/\lambda < 1$ as only for such a situation the use of periodic arrays as frequency and polarization fulfi the condition for the fundamental floque mode $L = 0$ is propagating in this range. The designed wavelength λ_0 is taken to be $1.55\mu m$. The effect of the transmission spectrum for a triangular lattice with $H = 1.0\lambda_0$ and $d_1 = 0.5d$ as the number of layers is increasing is shown in Fig.2. The parameters used are indicated in the figur and multi stop band formation was noticeable within the indicated frequency range. This region is where there is no transmission across the multilayered structure. As the number of layers increases, the stop bands are flatte at the top and the linewidth are sharpen when the number of layers is 20.

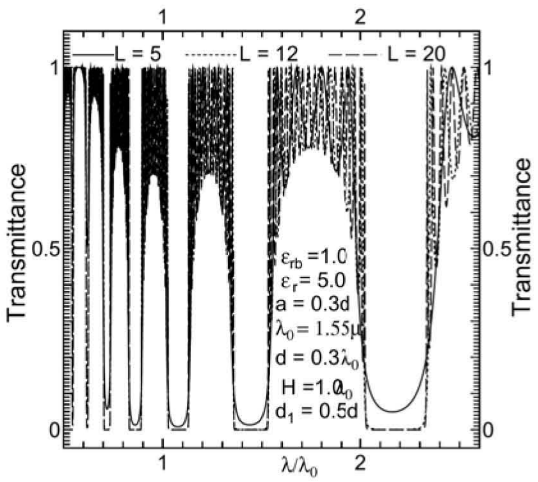

図 2: Transmittance of fundamental space harmonics vs normalized wavelength λ/λ_0 parameters: $a = 0.3d$, $\varepsilon_r = 5$.

Bearing in mind the results of the previous example, the reflectio coefficient for 20 layers is analysed as a function of the normalized wavelength at two different layer spacing $H = 0.3\lambda_0$ and $H = 0.5\lambda_0$ illustrated by the solid and dot-lines respectively as shown in Fig.3. It is observed that at a particular frequency, the two stop bands meet and this peculiar property is very useful in the design of frequency selection filters The frequency can be use as a transmission window at the stop band region if the structures are cascaded together to form one multilayered structure. The numerical results in Fig.4 show the case where in the two multilayered structures of Fig.3 are stacked together. It can be seen that within electro-

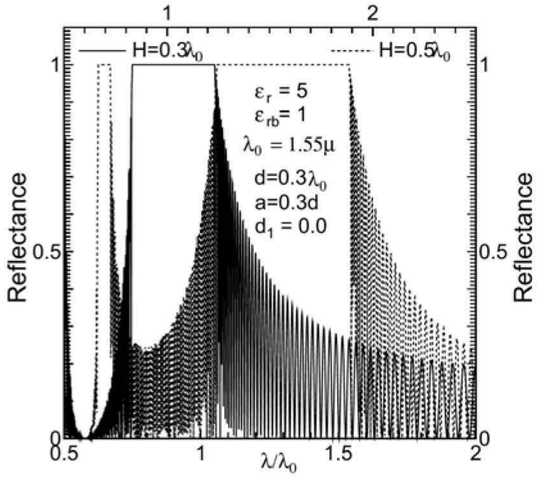

図 3: Reflectanc vs normalized wavelength λ/λ_0 parameters: $a = 0.3d$, $\varepsilon_r = 5$, $d = 0.3\lambda_0$.

magnetic stop band region $0.747 < \lambda/\lambda_0 < 1.541$, a microcavity effect is obtained at $\lambda = 1.058\lambda_0$ this is a transmission window which is observed when the upper structure of 20 layers with separation $H = 0.3\lambda_0$ and the lower structure of 40 layers with separation $H = 0.5\lambda_0$ are cascaded to form a 60 multilayered structures. A transmission peak of 0.3 is obtained is transmitted at this frequency. The transmission energy can be increase at

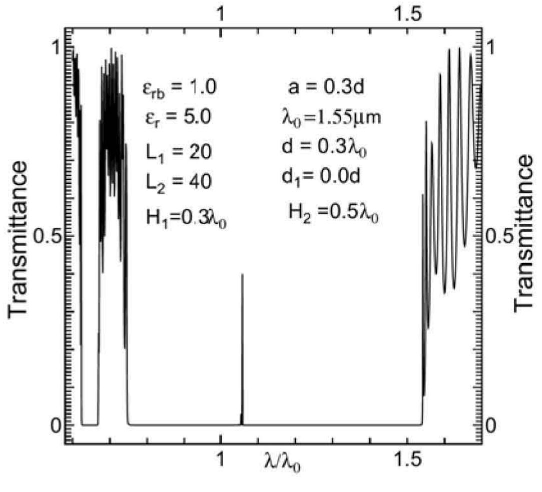

図 4: Transmittance vs normalized wavelength λ/λ_0 for two DBRs layer spacing of $H_1 = 0.3\lambda_0$ and $H_2 = 0.5\lambda_0$.

this frequency by changing the stacking configuratio as shown in Fig.5. in which three DBR structures are cascaded. In this case, a 40 layers with spacing $H_2 = 0.7\lambda_0$ is sandwiched between a 20 layers and 60 layers structures of Fig.4. For such a structure, the transmission window

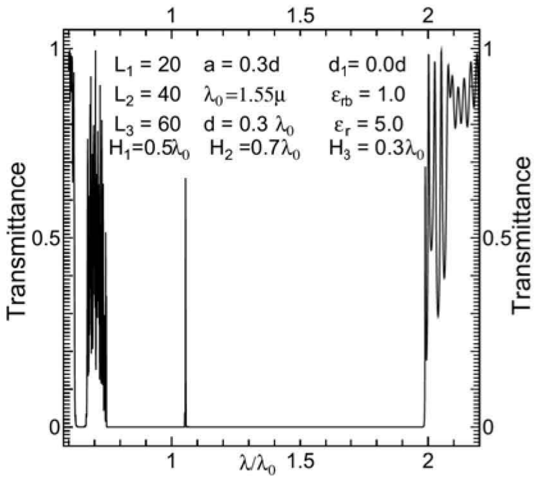

図 5: Transmittance vs normalized wavelength λ/λ_0 for three DBRs $H_1 = 0.3\lambda_0$, $H_2 = 0.7\lambda_0$ and $H_3 = 0.5\lambda_0$.

occurs at the same frequency but the transmittance energy is increased to 0.65 and the stop band width occurs at $0.747 < \lambda/\lambda_0 < 1.987$ with an increased in bandwidth of $0.446\lambda_0$ from the bandwidth of the structure in *Fig*.4. This spectra property is used in the designed of WDM wherein single wavelength is transmitted within the stop band region.

4. Conclusion

The Transmittance of TM wave from a multilayered periodic structures has been analyzed using Moment Method and TMM. The stability of the matrix computation for large number of layers especially at higher frequency range was attained using eigenvalues decomposition method. This enables us to clear identify the formation of stopband. The effect of transmittance the cascaded multilayerd structures with different layer spacing is shown. When two stop bands meet at a particular frequency, such frequency can be transmitted when the structures are stack together. As such, electromagnetic tunneling effect was obtained whose transmission energy is increased by stacking more multilayered structures.

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