Dispersion Characteristics for a Chiral Slab Waveguide

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Abstract

We examine the dispersion characteristics for a chiral slab waveguide. The waveguide consists of the homogeneous chiral media in the film and cladding. The electromagnetic fields are decomposed into the right- and left-handed circularly polarised fields. The guided mode expressions are derived by the wave equations and the solutions in the film and the cladding are obtained. Using the boundary condition at the interfaces yields the eigenvalue equation for the hybrid guided modes. The dispersion relation is examined numerically.

Key Words :
Chiral medium, Waveguide, Dispersion characteristics, Thin film

1. Introduction

The dispersion relation is so important for the waveguide design. So far, the microwave waveguide and the optical waveguide have been analyzed from the analytical, numerical, and experimental points of view. The waveguide including the chiral medium has been also studied [1)-8)]. As is well known, the propagation guided mode for the chiral waveguide is hybrid mode since the electric and magnetic fields are coupled each other due to the chirality parameter of the constitutive relation.

In this article, the dispersion characteristics for a chiral slab waveguide are examined. The electromagnetic fields are decomposed into the right-handed circularly polarised (RCP) and left-handed circularly polarised (LCP) fields. The guided mode analyzed is the symmetric even mode. The odd mode can be analyzed in the same way. The eigenvalue equation for the chiral slab waveguide is obtained by the boundary conditions at the interfaces. As numerical examples, the dispersion curves are shown when a film and/or a cladding are chiral media. The time factor $\exp(j\omega t)$ is suppressed throughout the manuscript.

2. Fields representation and eigenvalue equation

Consider a symmetric chiral slab waveguide with thickness $2d$ as shown in Fig. 1. It consists of a thin chiral dielectric film bounded by semi-infinite chiral dielectric cladding. The relative permittivity, relative permeability, and chirality parameter in the film and cladding are ε_i, μ_i and κ_i ($i = 1$ or 2), respectively.

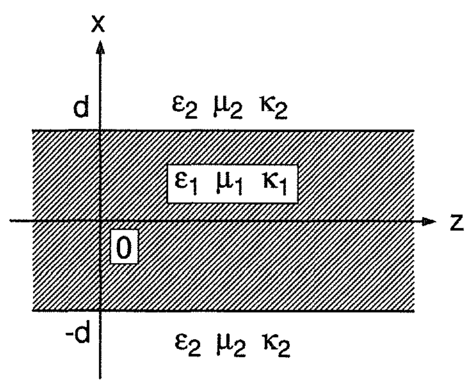

Figure1. Geometry of chiral slab waveguide.

The constitutive relations for the chiral media in this paper are given by

$$\mathbf{D} = \varepsilon_0\varepsilon_i\mathbf{E} - j\kappa_i\sqrt{\mu_0\varepsilon_0}\,\mathbf{H} \tag{1}$$

$$\mathbf{B} = j\kappa_i\sqrt{\mu_0\varepsilon_0}\,\mathbf{E} + \mu_0\mu_i\mathbf{H}\,, \quad (i = 1 \text{ or } 2) \tag{2}$$

The electric and magnetic fields are expressed in terms of the following form:

$$\mathbf{E} = \mathbf{E}^+ + \mathbf{E}^-\,, \qquad \mathbf{H} = j(\mathbf{E}^+ - \mathbf{E}^-)/\eta \tag{3}$$

where η is the wave impedance in the media. The "+" and "−" symbols correspond to the RCP and LCP waves in the chiral media, respectively. Substitution Eq. (3) into the Maxwell's equation leads the following equation:

$$\nabla \times \mathbf{E}^\pm = \pm k_\pm \mathbf{E}^\pm \tag{4}$$

It is assumed that the z dependence of the fields is $\exp(j\beta z)$.

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From Eq. (4), the wave equations for the RCP and LCP fields can be obtained

$$\frac{d^2E_y^{\pm}}{dx^2}+(k_{\pm,i}^2-\beta^2)E_y^{\pm}=0 \qquad (5)$$

The y component of the electric field of the guided even mode is expressed by

$$E_y=\begin{cases}A^{+}\exp\{-\gamma_{+}(|x|-d)\} & \\ \quad +A^{-}\exp\{-\gamma_{-}(|x|-d)\} & |x|>d \\ B^{+}\cos\delta_{+}d+B^{-}\cos\delta_{-}d & |x|<d\end{cases} \qquad (6)$$

where

$$\gamma_{\pm}=\sqrt{\beta^2-k_{\pm,2}^2} \qquad (7)$$

$$\delta_{\pm}=\sqrt{k_{\pm,1}^2-\beta^2} \qquad (8)$$

and $k_{\pm,i}=k_0(\sqrt{\varepsilon_i\mu_i}\pm\kappa_i)$ and k_0 is the wavenumber in free space. The other components of the electromagnetic fields are obtained using Eqs. (3) and (4). $A^{\pm}$ and $B^{\pm}$ are unknown coefficients to be determined from the boundary conditions at the interfaces. The eigenvalue equation is derived as follows:

$$-\frac{\gamma_{-}}{\eta_1^2k_{-,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d+\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$+\frac{2\delta_{+}\delta_{-}}{\eta_1\eta_2k_{+,1}k_{-,1}}\sin\delta_{+}d\sin\delta_{-}d$$
$$-\frac{\gamma_{-}}{\eta_1\eta_2k_{-,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d-\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$-\frac{\gamma_{+}}{\eta_1^2k_{+,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d+\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$+\frac{\gamma_{+}}{\eta_1\eta_2k_{+,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d-\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$+\frac{2\gamma_{+}\gamma_{-}}{\eta_1\eta_2k_{+,2}k_{-,2}}\cos\delta_{+}d\cos\delta_{-}d$$
$$+\frac{2\delta_{+}\delta_{-}}{\eta_1\eta_2k_{+,1}k_{-,1}}\sin\delta_{+}d\sin\delta_{-}d$$
$$-\frac{\gamma_{-}}{\eta_1\eta_2k_{-,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d-\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$-\frac{\gamma_{-}}{\eta_2^2k_{-,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d+\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d\right]$$
$$+\frac{\gamma_{+}}{\eta_1\eta_2k_{+,2}}\left[\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d-\frac{\delta_{-}}{k_{-,1}}\sin\delta_{-}d\cos\delta_{+}d\right]$$
$$+\frac{2\gamma_{+}\gamma_{-}}{\eta_1\eta_2k_{+,2}k_{-,2}}\cos\delta_{+}d\cos\delta_{-}d$$
$$-\frac{\gamma_{+}}{\eta_2^2k_{+,2}}\left[\frac{\delta_{-}}{k_{-,1}}\cos\delta_{+}d\sin\delta_{-}d+\frac{\delta_{+}}{k_{+,1}}\sin\delta_{+}d\cos\delta_{-}d\right]$$
$$=0 \qquad (9)$$

The cutoff frequency can be obtained by setting $\gamma_{+}=0$ in Eq. (9) and the following transcendental equation is obtained.

$$4\frac{\delta_{+}\delta_{-}}{\eta_1\eta_2k_{-,1}k_{+,2}}\tan\delta_{+}d\tan\delta_{-}d$$
$$-\frac{\delta_{+}}{k_{+,1}}\frac{\gamma_{-}}{k_{-,2}}\frac{\eta_1^2+\eta_2^2}{(\eta_1\eta_2)^2}\tan\delta_{+}d$$
$$-\frac{\delta_{-}}{k_{-,1}}\frac{\gamma_{-}}{k_{-,2}}\frac{\eta_1^2-\eta_2^2}{(\eta_1\eta_2)^2}\tan\delta_{-}d=0 \qquad (10)$$

When the cladding is the achiral medium ($\kappa_2=0$), γ_{-} becomes null in Eq. (10). The normalized cutoff frequency is obtained as follows:

$$V_{c,\pm}^{m}=m\pi\sqrt{\frac{\varepsilon_1-\varepsilon_2}{(\varepsilon_1\mu_1\pm\kappa_1)^2-(\varepsilon_2\mu_2)^2}} \quad m=0,1,2,\cdots \qquad (11)$$

This result is the same as that in the reference[4)]. When the cladding is the chiral medium, we have to solve the transcendental equation (10) to obtain the cutoff frequency.

The eigenvalue equation for the odd guided mode can be obtained when $\cos\delta_{\pm}d\to\sin\delta_{\pm}d$ and $\sin\delta_{\pm}d\to-\cos\delta_{\pm}d$ in Eq. (9).

Eq. (9) is solved using the tool such as the Newton method numerically to obtain the propagation constant in z direction.

3. Numerical results

At first, we compare the results with those in Reference[4)]. The relative permeability μ_1 and μ_2 are assumed to be 1.0 throughout the paper. The parameters are set up with $\sqrt{\varepsilon_1}=n_1=2.1$, $\sqrt{\varepsilon_2}=n_2=2.0$, $\kappa_1=0.01$. Figure 2 shows the dispersion curves for the chiral slab waveguide where the film is only the chiral media ($\kappa_2=0$). $V(=k_0d\{n_1^2-n_2^2\}^{1/2})$ is the normalized frequency, and the symbol H_0 in the figure means the hybrid mode of the lowest-order. It is found that the eigenvalue of the RCP fields approaches $n_1+\kappa_1$ and that of the LCP fields approaches $n_1-\kappa_2$. This is due to Eq. (7). The range of the eigenvalue of the even guided mode for the RCP and LCP fields is

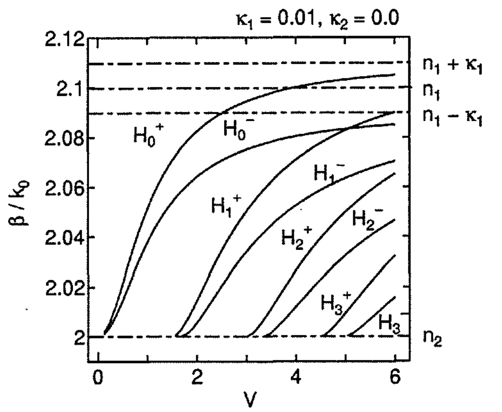
$$n_2<\beta/k_0<n_1+\kappa_1 \qquad \text{RCP field} \qquad (12)$$

$$n_2<\beta/k_0<n_1-\kappa_1 \qquad \text{LCP field} \qquad (13)$$

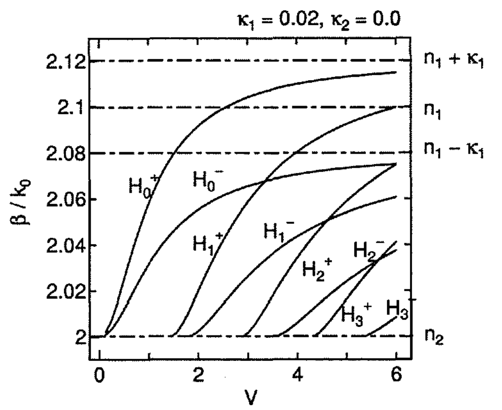
The cutoff frequency of the higher modes separate each other as the chiral parameter increases [4)].

Figure 3 shows the dispersion curves for the chiral cladding with a achiral film. As is pointed out in reference [7)], it is seen that the single mode can be propagated at the lower frequency.

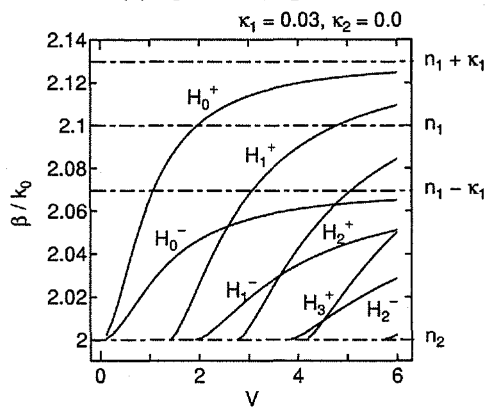
Figure 4 shows the dispersion curves for the chiral slab waveguide where both the film and cladding are the chiral



(a) $\kappa_1 = 0.01,\ \kappa_2 = 0$

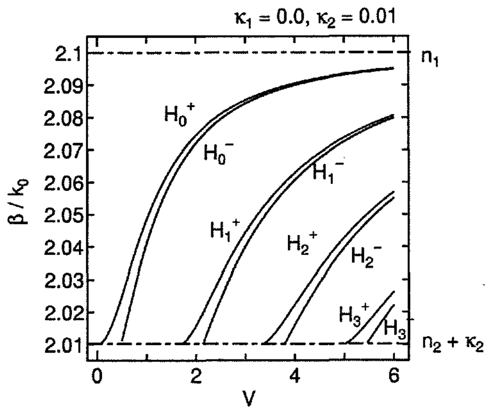


(b) $\kappa_1 = 0.02,\ \kappa_2 = 0$

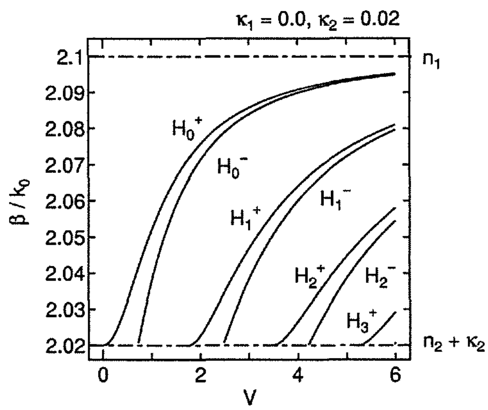


(c) $\kappa_1 = 0.03,\ \kappa_2 = 0$

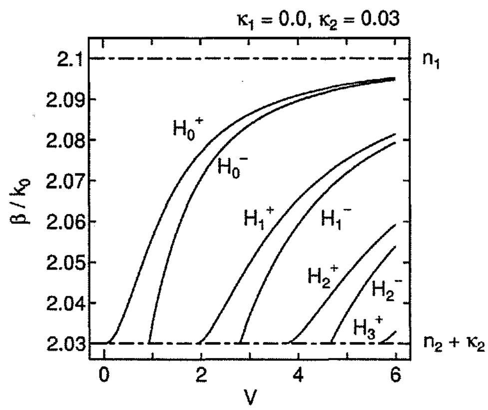
Figure2. Dispersion curves for the slab waveguide where only the film is the chiral media.



(a) $\kappa_1 = 0,\ \kappa_2 = 0.01$



(b) $\kappa_1 = 0,\ \kappa_2 = 0.02$



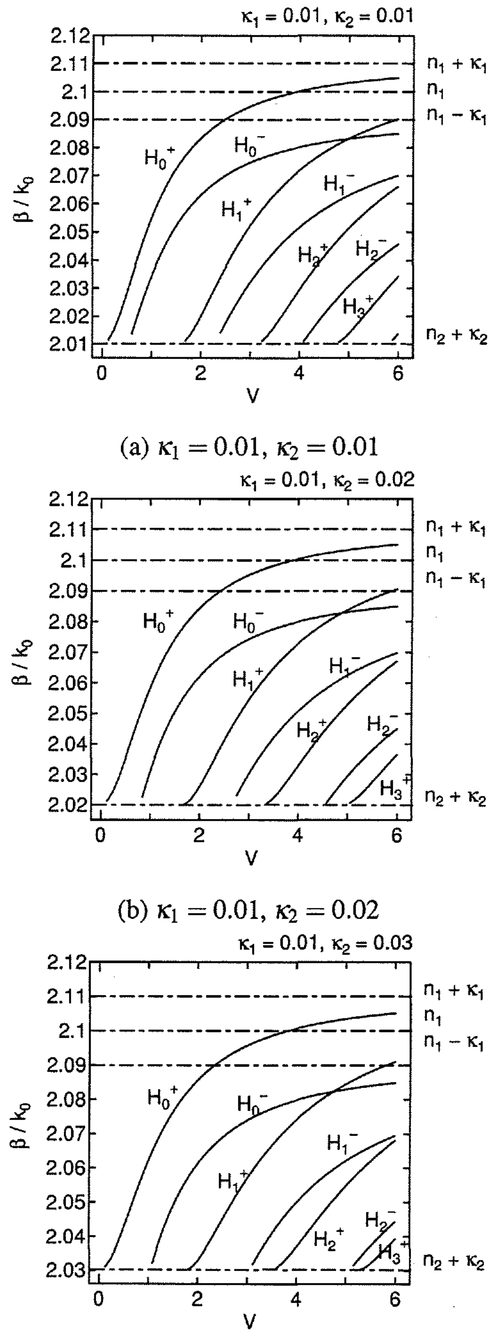
(c) $\kappa_1 = 0,\ \kappa_2 = 0.03$

Figure3. Dispersion curves for the slab waveguide for the chiral cladding with an achiral film.

media. The cutoff frequencies of the LCP and RCP fields for the $H_0^{\pm}$ mode are separated although the eigenvalue for the guided mode of the chiral film with the achiral cladding is the same as shown in Fig. 2. In the range of $n_2 - \kappa_2 < \beta/k_0 < n_2 + \kappa_2$, the eigenvalue can't exist since the LCP field is the guided mode and the RCP field is the radiation mode. The waveguide using the chiral cladding with the achiral film can support the single mode operation [7]. The waveguide for the chiral film with the chiral cladding has the same property and the frequency range for the single mode operation is wider than that of Fig. 3.

4. Conclusions

The dispersion relation for a chiral slab waveguide consisted of the chiral media in the film and cladding has been examined. It has been shown that the cutoff frequency depends on the chiral parameter and for higher frequency, the eigenvalue approaches to the different values from the achiral waveguide. Also, it has been found that the waveguide



(a) $\kappa_1 = 0.01$, $\kappa_2 = 0.01$

(b) $\kappa_1 = 0.01$, $\kappa_2 = 0.02$

(c) $\kappa_1 = 0.01$, $\kappa_2 = 0.03$

Figure4. Dispersion curves for the slab waveguide where both the film and cladding are the chiral media.

for the chiral film with the chiral cladding can support the single mode operation. In the future, the design problem of the chiral waveguide will be considered.

Acknowledgement

The author greatly appreciates the contribution for the numerical calculation of Mr. Yamanaka.

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