Two-Dimensional Input Tapes with One-Counter Languages Not Accepted by Deterministic Rebound Automata

Makoto SAKAMOTO[1)], Hiroaki KAWANO[2)], and Makoto SAITO[2)]

Abstract

M.Blum and C.Hewitt first proposed two-dimensional automata as a computational model of two-dimensional pattern processing, and investigated their pattern recognition abilities[1]. Since then, many researchers have been investigating a lot of properties about automata on a two-dimensional tape. However, there are a lot more open problems. For instance, it was unknown whether there exists a language accepted by a two-way nondeterministic one counter automaton, but not accepted by any deterministic rebound automaton. In this paper, we try to solve this problem, and show that there exists such a language.

Keywords: *nondeterminism, one counter automaton, rebound automaton, two-dimensional tape, chunk*

1 Introduction and Preliminaries

Two-dimentional automata were first proposed and investigated their pattern recognition abilities in 1967[1]. Since then, many researchers in this field have been investigating a lot of properties about two-dimensional automata[5]. In Ref.[6], a new type of language acceptor, called the rebound automaton, was proposed and its accepting power was investigated by Sugata, Umeo, and Morita. A rebound automaton has the same structure as a two-dimensional finite automaton[1], but an input to it is a square tape whose top row is a word to be recognized, and whose other symbols are all blank.It is demonstrated in Ref.[6] that rebound automata have some kind of counting ability, and thus they can accept many non-regular languages. But it is unknown whether there exists a language accepted by a two-way nondeterministic one counter automaton[3], but not accepted by any nondeterministic rebound automaton. This paper solves this problem, and shows that there exists such a language. This result implies that the counting ability of nondeterministic rebound automata are not sufficient to simulate the counting ability of two-way nondeterministic one counter automata.

Let Γ be a finite set of symbols. A two-dimensional tape over Γ is a two-dimensional rectangular array of elements of Γ.The set f all two-dimensional tapes over Γ is denoted by $\Gamma^{(2)}$.

Given a tape $x \in \Gamma^{(2)}$, we let $l_1(x)$ be the number of rows of x, and $l_2(x)$ be the number of columns of x. If $1 \leq i \leq l_1(x)$ and $1 \leq j \leq l_2(x)$, we let $x(i,j)$ denote the symbol in x with coordinates (i,j).

Futhermore, we define $x[(i,j),(i',j')]$, only when $1 \leq i \leq i' \leq l_1(x)$ and $1 \leq j \leq j' \leq l_2(x)$, as the two-dimensional tape z satisfying the following :

(i) $l_1(z)=i'-i+1$ and $l_2(z)=j'-j+1$;

(ii) for each $k,r(1 \leq k \leq l_1(z), 1 \leq r \leq l_2(z))$, $z(k,r)=$

1) Associate Professor, Dept. of Computer Science and Systems Engineering, Miyazaki University

2) Undergraduate Student, Dept. of Computer Science and Systems Engineering, Miyazaki University

$x(k+i-1, r+j-1)$.

A deterministic rebound automaton (DRA) is a system $M = (K, \Sigma, \sharp, B, q_0, \Delta, \delta, F)$, where K is a finite set of state, Σ is a finite set of input symbols, $\sharp$ is the blank symbol (not in Σ), B is the boundary symbol(not in Σ), $q_0(\in K)$ is the start states, $F(\subseteq K)$ is a set of accepting states, $\delta : K \times (\Sigma \cup \{\sharp, B\}) \to K \times \Delta$ is the control function, and $\Delta = \{L, R, U, D\}$ can be thought of as a set of directions (left, right, up, down).

An input tape for M is a two-dimentional square tape over $\Sigma \cup \{\sharp\}$ surrounded by the boundary symbol B, whose top row is a word $a_1 a_2 \cdots a_n \in \Sigma^+ (n \geq 1)$, and whose other symbols are all $\sharp'$s (See Fig.1.). $\delta(q, a) \ni (p, d)$ means that if M reads the symbol a in state q, it can enter state p and move in direction d. Suppose that an input tape x (as shown in Fig.1.) whose top row is a word $w = a_1 a_2 \cdots a_n \in \Sigma^+ (n \geq 1)$ is presented to M. Mstarts in state q_0 on the upper left-hand corner of x. If M falls off the tape x, M can make no further move. We say that the word w (which is the top row of x) is accepted by M if M eventually enters an accepting state somewhere on x. We denote the set of words accepted by M by $T(M)$.

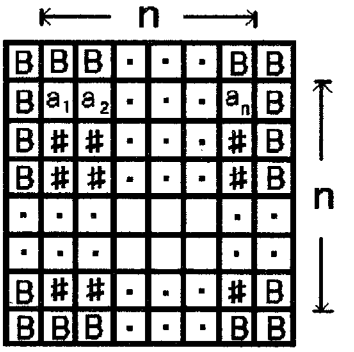

Fig. 1: An input tape to DRA.

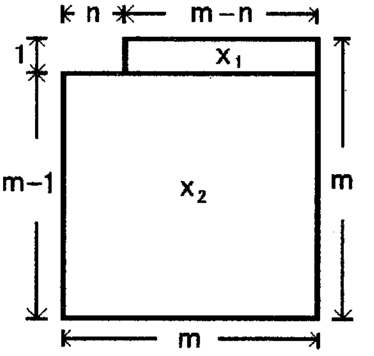

Fig. 2: An(m, n)−chunk.

2 Results

Here we give a preliminary result which is used to prove our main theorem.

For each $m \geq 2$ and each $1 \leq n \leq m-1$, an (m, n)−chunk is a pattern (over $\{0,1,2,\flat,\sharp\}$) as shown in Fig.2, where $x_1 \in \{0,1,2,\flat\}^{(2)}$, $x_2 \in \{\sharp\}^{(2)}$,$l_1(x_1) = 1$, $l_2(x_1) = m-n$, $l_1(x_2) = m-1$, and $l_2(x_2) = m$.

Let M be a DRA whose input alphabet is $\{0, 1, 2, \flat\}$, and $\sharp$ and B be the blank symbol and the boundary symbol of M, respectively. For any $(m, n) - chunk$ x, we denote by $x(B)$ the pattern (obtained from x by surrounding x with B's) shown in Fig.3.

Below, we asume without loss of generality that M enters or exits the pattern $x(B)$ only at the face designated by the bold line in Fig.3.Thus, the number of entrance points to $x(B)$ (or exit points from $x(B)$) for M is $n+3$.

We suppose that these entrance points (or exit points) are numbered $1, 2, \cdots, n+3$ in an appropriate way.

Let $P = \{1, 2, \cdots, n+3\}$ be the set of these entrance points (or exit points).

For each $i \in P$ and each $q \in K$ (K is the set of states of M), let $M_{(i,q)}(x(B))$ be a subset of $P \times K \cup \{L\}$ which is defined as follows (L is a new symbol) :

(i) $(j, p) \in M_{(i,q)}(x(B)) \Longleftrightarrow$

when M enters the pattern $x(B)$ in state q and at point i, it may eventually exit $x(B)$ in state p and at point j.

(ii) $L \in M_{(i,q)}(x(B)) \Longleftrightarrow$

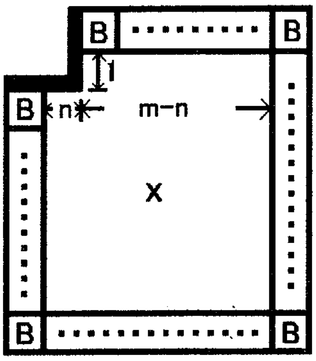

Fig. 3: $x(B)$.

when M enters the pattern $x(B)$ in state q and at point i, it may not exit $x(B)$ at all.

Let x, y be any two different$(m,n)-$chunks. We say that x and y are $M-$equivalent if for any $(i,q) \in P \times K, M_{(i,q)}(x(B)) = M_{(i,q)}(y(B))$. Thus, M cannot distinguish between two$(m,n)-$chunks which are $M-$equivalent.

Clealy, $M-$equivalence is an equivalence relation on $(m,n)-$chunks, and we get the following lemma.

[Lemma 1] *There are at most $((n+3)k+1)^{(n+3)k}M-$equivalence classes of $(m,n)-$chunks, where k is the number of state of M.*

(Proof) The proof is similar to that of Lemma 4.3 in Ref.[4]. □

Note that the number of M-equivalence class of $(m,n)-$chunks is independent of m.

Let $E = \{x_1 \flat x_2 \flat \cdots \flat x_k | k \geq 1$ and there is $l \geq 0$ such that $x_i \in \{0,1\}^l$ for $i = 1, \cdots, k\}$.

Let $s : \{0,1\}^* \to \{0,1,2\}^*$ be a function such that $s(a_1 \cdots a_l) = a_1 2^{l+2} a_2 2^{l+2} \cdots a_{l-1} 2^{2l-1}$, where $a_i \in \{0,1\}$ for $i = 1, \cdots, l$.

Further, let $h(x_1 \flat \cdots \flat x_k) = s(x_1) \flat \cdots \flat s(x_k)$ for each $x_1 \flat \cdots \flat x_k \in E$.

Then we define $L^h = \{x_0 \flat h(x_1 \flat \cdots \flat x_k) | k \geq 1$, $x_1 \flat \cdots \flat x_k \in E$, and there is $1 \leq j \leq k$ such that $x_j = x_0\}$.

We are now ready to prove our main theorem.

[Theorem 1] *There exists a language accepted by a two-way nondeterministic one counter automaton, but not accepted by any DRA. L_h is such a language.*

(Proof) It is shown in Ref.[2] that L_h is accepted by a two-way nondeterministic one counter automaton.

Below we show that L_h is not accepted by any DRA.

Suppose that L_h is accepted by some DRA M with k states. We can assume without loss of generality that when M accepts a word u in L_h, it enters an accepting state on the upper left-hand corner of the input tape whose top row is u, and that M never falls off an input tape out of the boundary symbol B.

For each $n \geq 1$, let

$V(n) = \{x_0 \flat h(x_1 \flat x_2 \flat \cdots \flat x_{f(n)}) | x_0 \{0,1\}^n$ and $\forall i (1 \leq i \leq f(n))[x_i \in \{0,1\}^n]\}$, where $f(n) = 2^n$;

$V'(n) = \{x \in \{0,1,2,\sharp,\flat\}^{(2)} | l_1(x) = l_2(x) = n + (1 - \frac{1}{2}n + \frac{3}{2}n^2)f(n)$ and $x[(1,1),(1,l_2(x))]$(i.e., the top row of x)$\in V(n)$ and $x[(2,1),(l_1(x),l_2(x))] \in \{\sharp\}^{(2)}\}$, where $\sharp$ is the blank symbol of M; and $Y(n) = \{0,1\}^n$.

Clearly $|Y(n)| = 2^n = f(n)$ (where for any set A, $|A|$ denotes the number of elements of A), and so we let $Y(n) = \{v_1, v_2, \cdots, v_{f(n)}\}$.

For each $n \geq 1$, let $S(n) = \{\text{word}(x) | x \in V'(n)\}$, where $\text{word}(x) = \{v_j \in Y(n) | v_j = v_i$ for some $i (1 \leq i \leq f(n))\}$ for each x in $V'(n)$ whose top row is $x_0 \flat h(x_1 \flat x_2 \flat \cdots \flat x_{f(n)})$ for some $x_0, x_1, x_2, \cdots, x_{f(n)}$ in $\{0,1\}^n$.

Clearly, $|S(n)| = \binom{f(n)}{1} + \binom{f(n)}{2} + \cdots + \binom{f(n)}{f(n)} = 2^{f(n)} - 1$.

Note that the set $\{p|$ for some x in $V'(n)$, p is the pattern obtained from x by cutting the part $x[(1,1),(1,n)]$ off$\}$ is the set of all $(n + (1 - (\frac{1}{2})n + (\frac{3}{2})n^2)f(n), n)-$chunks. By Lemma 1, there are at most $t(n) = ((n+3)k+1)^{(n+3)k}M-$equivalence classes of $(n + (1 - (\frac{1}{2})n + (\frac{3}{2})n^2)f(n), n)-$chunks.

We denote these $M-$equivalence classes by $C_1, C_2, \cdots, C_{t(n)}$. For large n, $|S(n)| > t(n)$. For such a large n, there must be some $l, l' (l \neq l')$ in $S(n)$ and some $C_i (1 \leq i \leq t(n))$ such that the following statement holds:

"There exist two tapes x and y in $V'(n)$ such that

(i) for some word v in l but not in l', $x[(1,1),(1,n)] = y[(1,1),(1,n)] = v$,

(ii) word$(x) = l$ and word$(y) = l'$, and

(iii) both p_x and p_y are in C_i, where $p_x(p_y)$ is the $(n+(1-(\frac{1}{2})n+(\frac{3}{2})n^2)f(n), n)$−chunks obtained from x (from y) by cutting the part $x[(1,1),(1,n)]$ (the part $y[(1,1),(1,n)]$) off."

As is easily seen, the top row of x is in L^h, and so it is accepted by M. It follows that the top row of y is also accepted by M, which is a contradiction. (Note that the top row of y is not in L^h.)

This completes the proof of the theorem. □

3 Conclusion

We showed that there exists a language accepted by a two-way nondeterministic one counter automaton, but not accepted by any deterministic rebound automaton. It is still unknown whether there exists a language accepted by a two-way deterministic one counter automaton, but not accepted by any deterministic (or nodeterministic) rebound automaton.

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