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On the Norm Kernel for the $^{16}O+^{16}O+\alpha$ Cluster System

Toshimi SAKUDA

Abstract

A calculation method is presented for determining the norm kernel on the harmonic oscillator basis and is applied to the $^{16}O+^{16}O+\alpha$ three-cluster system. Eigenvalues and eigenstates of the norm kernel are calculated. Spectroscopic factors are discussed for $^{16}O+^{20}N$ and $\alpha+^{32}S$ channels with respect to the structure of the superdeformed band in the ^{36}Ar nucleus.

1. Introduction

Much attention has been devoted to superdeformed (SD) rotational bands in ^{36}Ar–^{48}Cr nuclei [1,2]. The collective aspects of these rotational bands are particularly interesting subjects for nuclear structure studies. I have performed α-cluster model calculations for ^{36}Ar–^{48}Cr and have shown that the α-cluster structure is a stable feature in this region [3]. In the case of ^{32}S and ^{36}Ar, the SD bands have been predicted by cranked Skyrme-Hartree-Fock (SHF) calculations [4,5]. The relation between the SD bands and the ^{16}O-cluster structure has also been discussed in several studies [4–7], largely because the SD structure of these nuclei is regarded as a key to understanding the relation between the SD bands and the cluster structure. Therefore, it would be interesting to apply the multi-cluster model approach, which can treat both the α- and ^{16}O-cluster configurations, to the SD band in ^{36}Ar. I also aim to clarify the relation between the α- and ^{16}O-cluster structures, as this will provide valuable insight into the SD bands and their strong collectivity.

2. The norm kernel

In the case of a system composed of three clusters, the model space is described by the set of wave functions

$$\Phi_{J(N23\,l23,\,N1l1)} = \sqrt{\frac{4!16!6!}{36!}}\mathcal{A}\left\{\phi^{\rm int}(\alpha)\phi^{\rm int}(^{16}{\rm O})\phi^{\rm int}(^{16}{\rm O})\left[U_{N_{23},l_{23}}(\mathbf{r}),\ U_{N_1,l_1}(\mathbf{R})\right]_J\right\}, \qquad (1)$$

where $\phi^{\rm int}$'s are the antisymmetrized internal wave functions and $U_{N23,\,l23}(\boldsymbol{r})$ and $U_{N1,\,l1}(\boldsymbol{R})$ are harmonic oscillator (HO) wave functions with $N_{23}l_{23}$ and N_1l_1 quantum numbers for the

relative motions. The relative coordinates $\boldsymbol{r}$ and $\boldsymbol{R}$ are shown in Fig. 1. We adopt a common oscillator constant a for all clusters. The model wave function is generated as a direct product of the two relative wave functions : $(N_{23},0)\times(N_1,0)$. The Pauli-allowed states are obtained by diagonalizing the norm kernel.

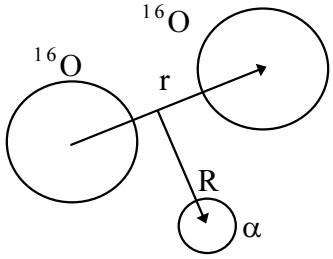

Fig. 1. Coordinates of the ^{16}O+^{16}O+α system.

$$\Sigma_{N23'l23',\,N1'l1'}\Big\langle\phi^{\rm int}(\alpha)\phi^{\rm int}(^{16}{\rm O})\phi^{\rm int}(^{16}{\rm O})\Big[U_{N23,\,l23}(\mathbf{r}),\,U_{N1,\,l1}(\mathbf{R})\Big]_J\Big|$$
$$\mathcal{A}\Big\{\phi^{\rm int}(\alpha)\phi^{\rm int}(^{16}{\rm O})\phi^{\rm int}(^{16}{\rm O})\Big[U_{N23'l23'}(\mathbf{r}),\,U_{N1',\,l1'}(\mathbf{R})\Big]_J\Big\}\Big\rangle C^{JNQ}_{N23'l23',\,N1'l1'} \qquad (2)$$
$$=\mu^{NQ}C^{JNQ}_{N23\,l23,\,N1\,l1}.$$

Thus the totally antisymmetrized basis that satisfies the orthonormal condition is given by

$$\Phi^{JNQ}=\frac{1}{\sqrt{\mu^{NQ}}}\Sigma_{N23\,l23,\,N1\,l1}C^{JNQ}_{N23\,l23,\,N1\,l1}\Phi_{J(N23\,l23,\,N1\,l1)}. \qquad (3)$$

The eigenstates are classified by the total oscillator quanta $N=N_{23}+N_1$ and the SU(3) symmetry. The symbol Q denotes the SU(3) quantum number $(\lambda,\mu)K$.

The problem then is to calculate the norm kernel and the eigenvalues μ^{NQ}. It is an enormous and very tedious task to treat the antisymmetrization operator within the internal and relative coordinate system. Therefore the computation of the kernels in heavy systems is done by calculating the corresponding GCM kernels and transforming them. It can be done by noting the fact that the corresponding GCM kernel has a generating function for the kernel in Eq. (2).

3. The norm kernel in GCM space

For clarity, more general notation is used in this section. We treat the GCM wave function of the form

$$\Phi^{GCM}(\mathbf{S}_1,\mathbf{S}_2,\mathbf{S}_3)=\sqrt{\frac{{\rm A}_1!{\rm A}_2!{\rm A}_3!}{{\rm A}!}}\mathcal{A}\Big\{\phi({\rm A}_1,{\rm S}_1)\phi({\rm A}_2,{\rm S}_2)\phi({\rm A}_3,{\rm S}_3)\Big\}, \qquad (4)$$

where $\mathcal{A}$ is the antisymmetrizer which exchanges the nucleons belonging to different clusters, and A_i is the mass number of the cluster i and specifies the cluster. The total mass number is $A=A_1+A_2+A_3$. For the present system we simply take $A_1=A_2=16$ and $A_3=4$. The antisymmetrized cluster wave function $\phi(A_i,S_i)$ is assumed to be $(0s)^4$ for α or $(0s)^4(0p)^{12}$ for ^{16}O shell-model wave functions centered at S_i. Usually the center-of-mass (CM) point of the total system is chosen as the origin O; i.e., $A_1S_1+A_2S_2+A_3S_3=0$. Since $\phi(A_i,S_i)$ is a Slater determinant, $\Phi^{GCM}(S_1,S_2,S_3)$ is also a Slater determinant. This makes the calculation by GCM easy.

The shell-model wave function of each cluster can be separated into the internal and the CM wave functions.

$$\phi(A_i,\mathbf{S}_i)=\phi^{\mathrm{int}}(A_i)\left[\frac{aA_i}{\pi}\right]^{\frac{3}{4}}\exp\left\{-\frac{aA_i}{2}(\mathbf{X}_i-\mathbf{S}_i)^2\right\}, \tag{5}$$

where $\mathbf{X}_i$ is the CM coordinate of the cluster A_i. Substituting this equation into Eq. (4) gives

$$\Phi^{GCM}(\mathbf{S}_1,\mathbf{S}_2,\mathbf{S}_3)=\sqrt{\frac{A_1!A_2!A_3!}{A!}}\prod_{i=1}^{3}\left[\frac{aA_i}{\pi}\right]^{\frac{3}{4}}\mathcal{A}\left[\exp\left\{-\sum_{i=1}^{3}\frac{aA_i}{2}(\mathbf{X}_i-\mathbf{S}_i)^2\right\}\phi^{\mathrm{int}}(A_1)\phi^{\mathrm{int}}(A_2)\phi^{\mathrm{int}}(A_3)\right]. \tag{6}$$

In order to observe the relative motions of the clusters, it is convenient to use the inter-cluster relative coordinates ($\boldsymbol{r}$ and $\boldsymbol{R}$) and the total CM coordinate $\mathbf{X}_G$.

$$\begin{aligned}\Phi^{GCM}(\mathbf{d},\mathbf{D})&=\left[\frac{aA}{\pi}\right]^{\frac{3}{4}}\exp\left\{-\frac{aA}{2}\mathbf{X}_G^{\,2}\right\}\sqrt{\frac{A_1!A_2!A_3!}{A!}}\\&\mathcal{A}\left[\left[\frac{a_1}{\pi}\right]^{\frac{3}{4}}\exp\left\{-\frac{a_1}{2}(\mathbf{R}-\mathbf{D})^2\right\}\left[\frac{a_{23}}{\pi}\right]^{\frac{3}{4}}\exp\left\{-\frac{a_{23}}{2}(\mathbf{r}-\mathbf{d})^2\right\}\phi^{\mathrm{int}}(A_1)\phi^{\mathrm{int}}(A_2)\phi^{\mathrm{int}}(A_3)\right],\end{aligned} \tag{7}$$

where $a_1=\frac{A_1(A_2+A_3)}{A}a$, $a_{23}=\frac{A_2A_3}{A_2+A_3}a$ and the displacement parameters of the cluster centers are $\mathbf{d}=\mathbf{S}_2-\mathbf{S}_3$ and $\mathbf{D}=\mathbf{S}_1-\frac{A_2}{A_2+A_3}\mathbf{S}_2-\frac{A_3}{A_2+A_3}\mathbf{S}_3$. In this way, the dependence on $\mathbf{X}_G$ is factored out, and the GCM wave function is therefore a non-spurious wave function about the CM motion.

The GCM norm kernel can be evaluated analytically. The GCM kernel turns out to be given by

$$\begin{aligned}\left\langle\Phi^{GCM}(\mathbf{d},\mathbf{D})\middle|\Phi^{GCM}(\mathbf{d}',\mathbf{D}')\right\rangle&=\exp\left\{-\frac{a_1}{4}(D^2+D'^2)-\frac{a_{23}}{4}(d^2+d'^2)\right\}\\&\times|\mathbf{B}|^4\exp\left\{-\frac{aA_1^2}{2A}\mathbf{D}\cdot\mathbf{D}'-\frac{aA}{2}\left[\frac{A_2}{A_2+A_3}\right]^2\mathbf{d}\cdot\mathbf{d}'-\frac{aA_1A_2}{2(A_2+A_3)}(\mathbf{D}\cdot\mathbf{d}'+\mathbf{d}\cdot\mathbf{D}')\right\},\end{aligned} \tag{8}$$

where

$$
\begin{aligned}
|\mathbf{B}| = & \left\{\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^4-\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^2\varepsilon 0(\mathbf{d}\cdot\mathbf{d}')(\rho\mathbf{d}\cdot\mathbf{d}')^2\right\}\varepsilon 2(\mathbf{S}_1\cdot\mathbf{S}'_1)\\
& -\left\{\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^3+\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^2\varepsilon 0(\mathbf{d}\cdot\mathbf{d}')(\rho\mathbf{d}\cdot\mathbf{d}')\right\}\varepsilon 2(\mathbf{d}\cdot\mathbf{S}'_1)\varepsilon 2(\mathbf{S}_1\cdot\mathbf{d}')\\
& +\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^3(\rho\mathbf{S}_1\cdot\mathbf{d}')\varepsilon 1(\mathbf{S}_1\cdot\mathbf{d}')\varepsilon 2(\mathbf{d}\cdot\mathbf{S}'_1)+\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^3(\rho\mathbf{d}\cdot\mathbf{S}'_1)\varepsilon 1(\mathbf{d}\cdot\mathbf{S}'_1)\varepsilon 2(\mathbf{S}_1\cdot\mathbf{d}')\\
& +\Big[-\left\{\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^3-\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')\varepsilon 0(\mathbf{d}\cdot\mathbf{d}')(\rho\mathbf{d}\cdot\mathbf{d}')^2\right\}(\rho\mathbf{S}_1\cdot\mathbf{S}'_1)\\
& +\left\{\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')^2-\varepsilon 1(\mathbf{d}\cdot\mathbf{d}')\varepsilon 0(\mathbf{d}\cdot\mathbf{d}')(\rho\mathbf{d}\cdot\mathbf{d}')\right\}(\rho\mathbf{d}\cdot\mathbf{S}'_1)(\rho\mathbf{S}_1\cdot\mathbf{d}')\Big]\varepsilon 1(\mathbf{d}\cdot\mathbf{S}'_1)\varepsilon 1(\mathbf{S}_1\cdot\mathbf{d}'),
\end{aligned}
\tag{9}
$$

with $\rho=a/2$. The abbreviated notations $\varepsilon 0(x)\equiv\exp(\rho x)$, $\varepsilon 1(x)\equiv\exp(\rho x)-1$ and $\varepsilon 2(x)\equiv\exp(\rho x)-1-\rho x$ are used. The determinant $|\mathbf{B}|$ can be written as a sum of terms, each of which is a product of polynomial parts and Gaussian parts.

$$
\begin{aligned}
|\mathbf{B}| = & \sum_{i=1}^{\max} g(i)(\rho\mathbf{D}\cdot\mathbf{D}')^{k1(i)}(\rho\mathbf{D}\cdot\mathbf{d}')^{k2(i)}(\rho\mathbf{d}\cdot\mathbf{D}')^{k3(i)}(\rho\mathbf{d}\cdot\mathbf{d}')^{k4(i)}\\
& \times\exp\left\{\omega 1(i)\rho\mathbf{D}\cdot\mathbf{D}'+\omega 2(i)\rho\mathbf{D}\cdot\mathbf{d}'+\omega 3(i)\rho\mathbf{d}\cdot\mathbf{D}'+\omega 4(i)\rho\mathbf{d}\cdot\mathbf{d}'\right\}.
\end{aligned}
\tag{10}
$$

When this expression is substituted into Eq. (7), the following result is obtained.

$$
\begin{aligned}
&\left\langle\Phi^{GCM}(\mathbf{d},\mathbf{D})\middle|\Phi^{GCM}(\mathbf{d}',\mathbf{D}')\right\rangle=\exp\left\{\frac{a_{23}}{4}(d^2+d'^2)-\frac{a_1}{4}(D^2+D'^2)\right\}\sum_{i_1=1}^{\max}g(i_1)\sum_{i_2=1}^{\max}g(i_2)\sum_{i_3=1}^{\max}g(i_3)\sum_{i_4=1}^{\max}g(i_4)\\
&\times(\rho\mathbf{D}\cdot\mathbf{D}')^{k1(i_1)+k1(i_2)+k1(i_3)+k1(i_4)}(\rho\mathbf{D}\cdot\mathbf{d}')^{k2(i_1)+k2(i_2)+k2(i_3)+k2(i_4)}\\
&\times(\rho\mathbf{d}\cdot\mathbf{D}')^{k3(i_1)+k3(i_2)+k3(i_3)+k3(i_4)}(\rho\mathbf{d}\cdot\mathbf{d}')^{k4(i_1)+k4(i_2)+k4(i_3)+k4(i_4)}\\
&\times\exp\left\{\left(\omega 1(i_1)+\omega 1(i_2)+\omega 1(i_3)+\omega 1(i_4)\right)\rho\mathbf{D}\cdot\mathbf{D}'\right\}\exp\left\{\left(\omega 2(i_1)+\omega 2(i_2)+\omega 2(i_3)+\omega 2(i_4)\right)\rho\mathbf{D}\cdot\mathbf{d}'\right\}\\
&\times\exp\left\{\left(\omega 3(i_1)+\omega 3(i_2)+\omega 3(i_3)+\omega 3(i_4)\right)\rho\mathbf{d}\cdot\mathbf{D}'\right\}\exp\left\{\left(\omega 4(i_1)+\omega 4(i_2)+\omega 4(i_3)+\omega 4(i_4)\right)\rho\mathbf{d}\cdot\mathbf{d}'\right\}.
\end{aligned}
\tag{11}
$$

The GCM wave function with a definite angular-momentum can be written in terms of the projections of the relative angular-momenta l_1 and l_{23}. The integrand involves the multiplication of the scalar products of parameters **d** and **D** by the spherical harmonics. We are therefore able to calculate the integrations analytically.

$$
\Phi^{GCM}\left((l_{23},l_1)JM\right)=\sum_{m_{23},m_1}\left((l_{23},m_{23},l_1,m_1\,|JM)\int d\hat{\mathbf{D}}\int d\hat{\mathbf{d}}\,Y_{l_{23},m_{23}}(\hat{\mathbf{D}})\,Y_{l_1,m_1}(\hat{\mathbf{d}})\times\Phi^{GCM}(\mathbf{d},\mathbf{D}).\right. \tag{12}
$$

As the GCM wave function is a generating function of the HO wave functions, this GCM wave function is also expanded by the HO wave functions of Eq. (1).

$$\Phi^{GCM}\Big((l_{23,}l_1)JM\Big)=\sum_{N1}\sum_{N23}C_{N1l1}C_{N23l23}\Big[\frac{\sqrt{a_1}}{2}D\Big]^{N1}\Big[\frac{\sqrt{a_{23}}}{2}d\Big]^{N23}\exp\Big[-\frac{a_{23}N_{23}}{4}d^2-\frac{a_1N_1}{4}D^2\Big] \times\Phi_{J(N23\,l23,\,N1\,l1)}\Phi_{OS}(\boldsymbol{X}_G), \tag{13}$$

where the coefficient

$$C_{N1}=(-1)^n\sqrt{\frac{2^{n+l+2}\pi}{n!(2n+2l+1)!}}\ \text{and}\ N=2n+l. \tag{14}$$

We are now able to derive the relation between the norm kernel in the GCM wave functions of Eq. (11) and that in the HO basis of Eq. (1).

$$\Big\langle\Phi^{GCM}\Big((l_{23,}l_1)JM\Big)\Big|\Phi^{GCM}\Big((l'_{23,}l'_1)JM\Big)\Big\rangle=\Sigma_{N1,N23}\,\Sigma_{N1',N23'}C_{N1l1}C_{N23l23}C_{N1'l1'}C_{N23'l23'} \Big(\frac{\sqrt{a_1}}{2}\Big)^{N1+N1'}\Big(\frac{\sqrt{a_{23}}}{2}\Big)^{N23+N23'}\exp\Big\{-\frac{a_{23}}{4}(d^2+d'^2)-\frac{a_1}{4}(D^2+D'^2)\Big\}D^{N1}d^{N23}D'^{N1'}d'^{N23'} \tag{15}$$
$$\Big\langle\Phi_{J(N23l23,N1l1)}\Big|\Phi_{J(N23'l23',N1'l1')}\Big\rangle.$$

This relation gives the transformation procedure from the GCM kernel to the kernel in the HO basis. The matrix elements of the norm kernel are obtained as the coefficients of the powers of d and D. Thus we are able to solve the eigenvalue problem of the norm kernel, which gives all of the necessary quantities.

4. Eigenstates of the norm kernel

The calculated allowed states are listed in Table I, all of which are classified according to the SU(3) symmetry. The states with total quanta $N\leqq 31$ are not allowed as a matter of course. The lowest N=32 space contains the important $(sd)^{-8}(fp)^4$ shell-model state (16,8), which is an excited state. It is noted that the $^{16}O+^{16}O+\alpha$ model cannot describe the $(sd)^{-4}$ ground configuration of ^{36}Ar. The configurations with a larger value of N have the capacity to present the ^{16}O and α-cluster states.

Table I. The Pauli-allowed states of the $^{16}O+^{16}O+\alpha$ system.
They are classified by the SU(3) label (λ,μ) with the multiplicity n.

N	$(\lambda,\mu)^n$
32	(16,8)
33	(19,7) (17,8) (15,9)
34	(22,6) (20,7) $(18,8)^2$ (16,9) (14,10)
35	(25,5) (23,6) $(21,7)^2$ $(19,8)^2$ $(17,9)^2$ (15,10) (13,11)
36	(28,4) (26,5) $(24,6)^2$ $(22,7)^2$ $(20,8)^3$ $(18,9)^2$ $(16,10)^2$ (14,11) (12,12)
37	(31,3) (29,4) $(27,5)^2$ $(25,6)^2$ $(23,7)^3$ $(21,8)^3$ $(19,9)^3$ $(17,10)^2$ $(15,11)^2$ (13,12) (11,13)
38	(34,2) (32,3) $(30,4)^2$ $(28,5)^2$ $(26,6)^3$ $(24,7)^3$ $(22,8)^4$ $(20,9)^3$ $(18,10)^3$ $(16,11)^2$ $(14,12)^2$ (12,13) (10,14)
39	(37,1) (35,2) $(33,3)^2$ $(31,4)^2$ $(29,5)^3$ $(27,6)^3$ $(25,7)^4$ $(23,8)^4$ $(21,9)^4$ $(19,10)^3$ $(17,11)^3$ $(15,12)^2$ $(13,13)^2$ (11,14) (9,15)
40	(40,0) (38,1) $(36,2)^2$ $(34,3)^2$ $(32,4)^3$ $(30,5)^3$ $(28,6)^4$ $(26,7)^4$ $(24,8)^5$ $(22,9)^4$ $(20,10)^4$ $(18,11)^3$ $(16,12)^3$ $(14,13)^2$ $(12,14)^2$ (10,15) (8,16)
41	(41,0) $(39,1)^2$ $(37,2)^2$ $(35,3)^3$ $(33,4)^3$ $(31,5)^4$ $(29,6)^4$ $(27,7)^5$ $(25,8)^5$ $(23,9)^5$ $(21,10)^4$ $(19,11)^4$ $(17,12)^3$ $(15,13)^3$ $(13,14)^2$ $(11,15)^2$ (9,16) (7,17)

For comparison, the allowed states of the $\alpha+^{32}S$ system are listed in Table II. The $\alpha+^{32}S$ cluster structure also contributes to our understanding of the SD states. States with the total quanta $N\leqq 7$ are not allowed in this system. We can see that the $^{32}S+\alpha$ cluster model space contains many important shell-model states. The lowest quanta N=8 states are the $(sd)^{-4}$ (0,8), (2,4) and (4,0), which are known as the main configurations of the ground state of ^{36}Ar. The important core-excited states such as the $(sd)^{-6}(fp)^2$ N=10 (10,4) state and the $(sd)^{-8}(fp)^4$ N=12 (16,8) state are included in the present model space.

Table II. SU(3) classification of the Pauli-allowed states of the $\alpha+^{32}S$ system.

N	(λ,μ)
8	(4,0) (2,4) (0,8)
9	(7,2)(6,1)(5,0) (6,4)(5,3)(4,2)(3,1) (5,6)(4,5)(3,4)(2,3) (4,8)(3,7)(2,6)(1,5) (2,9)(1,8)(0,7)
10	(10,4)(9,3)(8,2)(7,1)(6,0) (9,6)(8,5)(7,4)(6,3)(5,2)(4,1) (8,8)(7,7)(6,6)(5,5)(4,4)(3,3)(2,2) (6,9)(5,8)(4,7)(3,6)(2,5)(1,4) (4,10)(3,9)(2,8)(1,7)(0,6)
11	(13,6)(12,5)(11,4)(10,3)(9,2)(8,1)(7,0) (12,8)(11,7)(10,6)(9,5)(8,4)(7,3)(6,2)(5,1)

11	(10,9)(9,8)(8,7)(7,6)(6,5)(5,4)(4,3)(3,2) (8,10)(7,9)(6,8)(5,7)(4,6)(3,5)(2,4)(1,3) (6,11)(5,10)(4,9)(3,8)(2,7)(1,6)(0,5)
12	(16,8)(15,7)(14,6)(13,5)(12,4)(11,3)(10,2)(9,1)(8,0) (14,9)(13,8)(12,7)(11,6)(10,5)(9,4)(8,3)(7,2)(6,1) (12,10)(11,9)(10,8)(9,7)(8,6)(7,5)(6,4)(5,3)(4,2) (10,11)(9,10)(8,9)(7,8)(6,7)(5,6)(4,5)(3,4)(2,3) (8,12)(7,11)(6,10)(5,9)(4,8)(3,7)(2,6)(1,5)(0,4)

The lowest N=32 (16,8) state of Table I is not the ground configuration, but a 4$\hbar\omega$ excited configuration of ^{36}Ar. That corresponds to the N=12 (16,8) state of the $\alpha+^{32}$S cluster system.

5. Spectroscopic Factors

The decay width is one of the most important physical quantities for identifying the cluster structure. We calculate the spectroscopic factors (S-factors) of the α, ^{16}O, ^{20}Ne and ^{32}S-channels for the typical eigenstates of the norm kernel. As an example, the reduced width amplitude (RWA) for the ^{16}O+ ^{20}Ne channel is defined by

$$ry_{JM}(l_{23},l_1) = \sqrt{\frac{36!}{16!20!}}\, r\left\langle \phi^{\mathrm{int}}(^{16}\mathrm{O})\left[\phi^{\mathrm{int}}_{l_{23}}(^{20}\mathrm{Ne}), Y_{l_1}(\hat{\mathbf{r}})\right]_{JM}\middle|\Psi_{JM}(^{36}\mathrm{Ar})\right\rangle. \tag{16}$$

Integrating the RWA with respect to r, we obtain the corresponding S-factor.

$$S_{JM}(l_{23},l_1) = \int_0^\infty \left\{ry_{JM}(l_{23},l_1)\right\}^2 dr. \tag{17}$$

The S-factors of the ^{32}S+α channels are calculated similarly.

We calculate the α and ^{16}O S-factors for the (16,8) K=0, J=0$^+$ state, which is believed to be the main configuration of the SD band in ^{36}Ar. The resulting $\alpha+^{32}$S S-factors for the (16,8) band are listed in Table III. The wave function of ^{32}S is taken here as the (24,0) state. The (24,0) state is expected to be a main component of the SD band and is largely the ^{16}O+^{16}O cluster state.

Table III. The α+^{32}S (24,0) spectroscopic factors for the (16,8) state of ^{36}Ar.

(l_{23}, l_1) J	(16,8) *K=0, J=0*
(0, 0) 0	0.0488
(2, 2) 0	0.0499
(4, 4) 0	0.0308
(6, 6) 0	0.0128
(8, 8) 0	0.0028

It was found that the (16,8) state has large *S*-factors for the α+^{32}S (0^+) and α+^{32}S (2^+) cluster channels.

The ^{16}O +^{20}Ne *S*-factors for the (16,8) *K*=0 band are listed in Table IV. The (8,0) state was taken for the wave function of ^{20}Ne, which is the dominant configuration of the ground state. The ^{16}O +^{20}Ne *S*-factors for the (16,8) band are not generally expected to be small. This result indicates that the (16,8) state has a variety of cluster structures containing not only the two-cluster structure but also the three-cluster structure.

Table IV. The ^{16}O+^{20}Ne(8,0) spectroscopic factors for the (16,8) state of ^{36}Ar.

(l_{23}, l_1) J	(16,8) *K=0, J=0*
(0, 0) 0	0.00100
(2, 2) 0	0.00102
(4, 4) 0	0.00063
(6, 6) 0	0.00026
(8, 8) 0	0.00006

The α+^{32}S *S*-factors for the (16,8) *K*=0 band are presented in Table V. In this case, the wave function of ^{32}S is taken to be the (4,8) state, which is the main configuration of the ground state of ^{32}S. The calculation of the α+^{32}S(4,8) *S*-factors is outside of the framework of the ^{16}O+^{16}O+α cluster model, because the ^{16}O+^{16}O cluster system cannot describe the ^{32}S(4,8) state. These *S*-factors are therefore calculated in the α+^{32}S cluster model. We are able to specify the large values for the S-factors in the α+^{32}S(0^+) and α+^{32}S(2^+) channels. This result could be interpreted to indicate the importance of the α-cluster structure in the SD band. It is of interest to investigate the structure of ^{36}Ar in the combined framework of the ^{16}O+^{16}O+a and α+^{32}S cluster configurations.

Table V. The $\alpha+^{32}S(4,8)$ spectroscopic factors for the (16,8) state of ^{36}Ar

$(K\ I,\ \ l)\ \ J$	(16,8) *K=0, J=0*
(0 0, 0) 0	0.0412
(0 2, 2) 0	0.0318
(0 4, 4) 0	0.0103
(0 6, 6) 0	0.0016
(0 8, 8) 0	0.0001

6. Summary

I have presented a calculation method for determining the norm kernel on the HO basis. Applying this method to the $^{16}O+^{16}O+\alpha$ three-cluster system, eigenvalues and eigenstates of the norm kernel were calculated. They are characterized by SU(3)-classification. Using the obtained norm kernel, the spectroscopic factors have been calculated. An illustrative calculation of the $^{16}O+^{20}N$ and $\alpha+^{32}S$ spectroscopic factors has been carried out for the (16,8) state of ^{36}Ar. It is shown that both of the α- and ^{16}O-cluster configurations play important roles in the SD band of the ^{36}Ar nucleus.

References

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