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MODELING OF WIND PROFILE GRATIFYING GIVEN POWER AND CROSS SPECTRA*

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Abstract. *Two methods are proposed for generating the random wind velocity whose power and cross spectral densities are specified to reflect the real world. The one is based on the linear dynamic system with white Gaussian noise processes as the seed of randomness; while the other is derived based on the spectral representation of a stationary process from the Wiener processes defined on the frequency domain. The windward pressures are calculated from the wind velocities through aerodynamic admittance. Sample runs of windward pressure are illustrated by simulation works.*
Keywords: Modeling, Wind profile, Power and cross spectra, Simulation

1. **Introduction.** Wind-excited vibration of tall structures such as high-rise buildings or slender towers may cause fatal structural failure, discomfort to occupants or malfunction of equipment; hence, it is of particular importance to investigate the active and/or passive control problem of such tall structures whose random vibrations are caused by wind- and/or seismic disturbances. In order to research the active/passive control problem in structural design, it is inevitable to generate artificially wind and/or seismic waves. These waves are used to show how well the structure withstands random disturbance loads. Nowadays, the generation of artificial waves is considered as one of distinct technologies. A useful mathematical model for generating seismic waves was proposed by one of the authors using chirp signals and its simulation studies were conducted to test whether the proposed model recovers well or not [1].

Especially, in wind engineering, the analysis seems to have attained maturity to a certain extent [2], [3]. However, despite of such a situation, the white noise process whose spectral density is constant over frequency domain is traditionally employed as a model of wind load, disregarded utterly the fact that the spectral density of the actual wind is not constant. The reason why such stationary white noise is used is based mainly on the fact that the optimal control theory is developed in the linear-quadratic-Gaussian (LQG) framework where the stationary random load is assumed [4], [5]. Needless to say, the

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model should generate the wind profile in such a way that it reflects the actual spectral density as thoroughly as possible.

The success of the control against random wind loads hinges largely on how well the artificial wind loads represent the principal characteristics of the power spectrum of the wind. This implies that in the design and/or stability analysis of the structure a simple and useful method of generating the wind profile whose power spectrum gratifies the real (i.e., experimental) one. In spite of the needs of the times, up to the present time there are very few researches on the generation of artificial wind loads by computers.

The difficulty in generating wind profiles by computers lies on the matter how we can realize the given spectrum. For example, based on the multi-dimensional autoregressive model, Iwatani [6] proposed a method of simulation for generating wind fluctuations which satisfy the given power and cross spectra. In this method a recursive form is investigated to avoid solving the resultant numerous simultaneous equations. As pointed in [6] both power and cross spectra depend deeply on the coefficient parameters of the multi-dimensional autoregressive model, so that the determination of the parameters are quite difficult to obtain wind profiles similar to actual ones. The topic concerning the computer generation of wind profile is still important.

In this paper, two mathematical models which generate wind profiles satisfying the specified power and cross spectra are proposed. The paper is organized as follows. In Section 2 the models of power spectral density and the cross spectrum are given based on the models due to von Kármán and Davenport. Two mathematical models of wind velocity are proposed in Section 3. The one is given as the output of the linear dynamic system whose inputs are assumed to be random white Gaussian noise processes, while the other model is given based on the modified form of the spectral representation of stationary random process by introducing the Wiener process on the frequency domain. In Section 4 the generation of windward pressure and its power spectral density are stated. The procedure to generate profiles of wind velocity and windward pressure is given in Section 5, and some discussions on the direct generation of windward pressure are done in Section 6. Simulation studies are given in Section 7, and the conclusions are stated in Section 8.

2. **Power Spectral Density of Wind Velocity.** Consider an N-story building structure subjected to random wind forces. It is custom to consider that the wind velocity is decomposed into an average wind velocity which varies along the height of the structure and a wind fluctuation component which is a stationary random process [2]. Consequently, the wind load can be considered to consist of a static load due to the average velocity and a dynamic load due to wind velocity fluctuations. For structural control, only the dynamic loads is considered [7].

Here, let z_i ($i = 1, 2, \cdots, N$) be the height of ith floor, and let $v_i(t)$ be the wind velocity fluctuation at height z_i. In the architecture community in Japan, the following power spectral density of the wind velocity $v_i(t)$ due to von Kármán is recommended and often employed [8], [9]:

$$S_{v_iv_i}(\omega) = \frac{0.238\,\sigma_{v_i}^2}{b_i\left\{1 + \left(\dfrac{\omega}{2\pi b_i}\right)^2\right\}^{5/6}}, \tag{1}$$

where $\sigma_{v_i}^2$ is the variance parameter of velocity fluctuation $v_i(t)$;

$$b_i = \frac{\bar{v}_i}{L(z_i)}, \quad L(z_i) = \frac{2\pi}{\omega_{\max}}\sqrt{1.5}\,z_i = \frac{\sqrt{1.5}}{0.006}\,z_i^{0.42};$$

$\bar{v}_i$ is the mean wind velocity at z_i and this is assumed to follow a power law

$$\bar{v}_i = V_g \left(\frac{z_i}{h_g} \right)^{\alpha}$$

in which h_g is the gradient height, V_g the average wind velocity at the gradient height, and the exponent α is a constant taking between 0.15 and 0.5.

Let $v(t) = [\, v_1(t), \cdots, v_N(t) \,]^T$ be the wind velocity vector, and let $S_v(\omega)$ be the $N \times N$-matrix spectral density of $v(t)$. The diagonal elements of $S_v(\omega)$ consists of $\{S_{v_i v_i}(\omega)\}$ given by (1); while the off-diagonal ones are cross spectral densities $\{S_{v_i v_k}(\omega)\}$.

Lack of the model of cross spectrum can be covered by the idea of coherency which is a measure of extent how two wind velocities $v_i(t)$ and $v_k(t)$ are correlated. Then the (i,k)-element of $S_v(\omega)$ is given by

$$[S_v(\omega)]_{ik} = \begin{cases} S_{v_i v_i}(\omega) & (i = k) \\ \sqrt{S_{v_i v_i}(\omega)}\sqrt{S_{v_k v_k}(\omega)}\, \mathrm{coh}_{ik}(\omega) & (i \neq k) \end{cases} \tag{2}$$

in which $\mathrm{coh}_{ik}(\omega)$ is the coherence function and the following form is often employed [10]:

$$\mathrm{coh}_{ik}(\omega) = \exp\left\{ -c \frac{\omega}{2\pi} \frac{|z_i - z_k|}{V_r} \right\}, \tag{3}$$

where c is the decay factor which is recommended experimentally to be selected as $c = 8$; and V_r is the reference mean wind velocity at 10 m above the ground. If we employ the model (3), it is noted that the second expression in the right-hand side of (2) covers all components of $S_v(\omega)$ because $\mathrm{coh}_{ii}(\omega) = 1$ for all i.

3. **Models of Wind Velocity.** In this paper, two kinds of mathematical models are proposed for the generation of (stationary) random wind velocity. The one is given by a linear system in which white noise processes are used as the seeds of randomness; while the other is derived based on the spectral representation of a stationary process.

3.1. **Wind velocity model based on linear model.** Given a set of mutually independent white Gaussian noise processes $\{\gamma_k(t)\}_{k=1,2,\cdots,m}$ $(m \leq N)$ with zero-means and variances $\{\sigma_k^2\}$, the wind velocity $v_i(t)$ at height z_i is assumed to be generated as a (stationary) output of a linear dynamic system with inputs $\{\gamma_k(t)\}_{k=1,2,\cdots,m}$,

$$\begin{aligned} v_i(t) &= \sum_{k=1}^{m} \int_{-\infty}^{t} h_{v_i\gamma_k}(t-\tau)\gamma_k(\tau)\, d\tau \\ &= \sum_{k=1}^{m} \int_{0}^{\infty} h_{v_i\gamma_k}(\tau)\gamma_k(t-\tau)\, d\tau, \end{aligned} \tag{4}$$

where $h_{v_i\gamma_k}(\tau)$ is the impulse response between $\gamma_k(t)$ and $v_i(t)$. In the vector form, it is given as

$$v(t) = \int_0^{\infty} H_{v\gamma}(\tau)\gamma(t-\tau)\, d\tau, \tag{5}$$

where $v(t) = [\, v_1(t), \cdots, v_N(t) \,]^T$, $\gamma(t) = [\, \gamma_1(t), \cdots, \gamma_m(t) \,]^T$ (T denotes transpose); and $H_{v\gamma}(\tau)$ is the $N \times m$-impulse response matrix, $H_{v\gamma}(\tau) = [\, h_{v_i\gamma_k}(\tau) \,]_{i=1,\cdots,N;\, k=1,\cdots,m}$.

Let $G_{v\gamma}(j\omega)$ $(j = \sqrt{-1})$ be the frequency response matrix (Fourier transformation of $H_{v\gamma}(\tau)$) of dimension $N \times m$. Then, we have the relation,

$$S_v(\omega) = G_{v\gamma}(j\omega)\, W\, G_{v\gamma}^{*}(j\omega), \tag{6}$$

where W is the diagonal matrix, $W = \mathrm{diag}\{\sigma_1^2, \cdots, \sigma_m^2\}$, such that $\mathcal{E}\{\gamma(t)\gamma^T(\tau)\} = W\,\delta(t-\tau)$ ($\delta(\cdot)$: Dirac delta function); and the asterisk denotes the complex conjugate transpose.

Here, decompose $S_v(\omega)$ as

$$S_v(\omega) = S_v^{\frac{1}{2}}(\omega)\, \Psi(j\omega)\, \Psi^*(j\omega)\, S_v^{\frac{1}{2}}(\omega) \tag{7}$$

in which $S_v^{\frac{1}{2}}(\omega)$ is the square root matrix of $S_v(\omega)$; and $\Psi(j\omega)$ is an introduced (random or nonrandom) $N \times m$-matrix-valued complex function such that $\Psi(j\omega)\, \Psi^*(j\omega) = I$ (unit matrix) for all ω.

Decomposing W as $W = W^{\frac{1}{2}}\, W^{\frac{1}{2}}$, we obtain from (6) and (7) the following relation:

$$S_v^{\frac{1}{2}}(\omega)\, \Psi(j\omega)\, \Psi^*(j\omega)\, S_v^{\frac{1}{2}}(\omega) = G_{v\gamma}(j\omega)\, W^{\frac{1}{2}}\, W^{\frac{1}{2}}\, G_{v\gamma}^*(j\omega) \tag{8}$$

from which follows that

$$S_v^{\frac{1}{2}}(\omega)\, \Psi(j\omega) = G_{v\gamma}(j\omega)\, W^{\frac{1}{2}}$$

or

$$G_{v\gamma}(j\omega) = S_v^{\frac{1}{2}}(\omega)\, \Psi(j\omega)\, W^{-\frac{1}{2}}. \tag{9}$$

Hence, the impulse response matrix $H_{v\gamma}(\tau)$ can be obtained by the inverse Fourier transform of $G_{v\gamma}(j\omega)$ as

$$H_{v\gamma}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G_{v\gamma}(j\omega)\, e^{j\omega\tau}\, d\omega. \tag{10}$$

Using this, the wind velocity profile can be computed by (5) using (9).

It should be noted here that the role of the function $\Psi(j\omega)$ is to reflect phase lags between stories. Specifically, we take, for instance, in the case of $m = N$ as

$$\Psi(j\omega) = \mathrm{diag}\{e^{-j\theta_1(\omega)}, \cdots, e^{-j\theta_N(\omega)}\}, \tag{11}$$

where $\{\theta_i(\omega)\}$ are random numbers with uniform distribution between 0 to 2π, independent mutually, or as

$$\Psi(j\omega) = \mathrm{diag}\{\psi_i(j\omega), \cdots, \psi_N(j\omega)\} \tag{12}$$

with

$$\psi_i(j\omega) = \exp\left\{-j\omega\, \frac{c_0}{2\pi}\, \frac{z_i}{V_r}\right\} \quad (c_0 : \text{const.}). \tag{13}$$

3.2. **Wind velocity model based on spectral representation.** The wind velocity $v(t)$ is assumed to be stationary in the wide sense, having its spectral density matrix (2) with (3). It is well-known that every stationary random process has the spectral representation of [11]

$$v(t) = \int_{-\infty}^{\infty} e^{j\omega t}\, d\tilde{w}(\omega) \tag{14}$$

in which $\tilde{w}(\omega)$ is an N-vector mutually independent orthogonal process (in $\omega \in (-\infty, \infty)$) with properties:

$$\mathcal{E}\{d\tilde{w}(\omega)\} = 0 \tag{15}$$

$$\mathcal{E}\{d\tilde{w}(\omega_1)\, d\tilde{w}^*(\omega_2)\} = \begin{cases} 0 & (\omega_1 \neq \omega_2) \\ S_v(\omega_1)\, d\omega_1 & (\omega_1 = \omega_2). \end{cases} \tag{16}$$

In order to generate the orthogonal process having its covariance matrix $S_v(\omega)$, in this paper, introduce an m-vector (real) standard Wiener process $w(\omega)$ *on the frequency domain*, i.e.,

$$\mathcal{E}\{dw(\omega)\} = 0 \tag{17}$$

$$\mathcal{E}\{dw(\omega_1)\, dw^T(\omega_2)\} = \begin{cases} 0 & (\omega_1 \neq \omega_2) \\ I\, d\omega_1 & (\omega_1 = \omega_2), \end{cases} \tag{18}$$

and let the orthogonal increment $\{d\tilde{w}(\omega)\}$ in (14) be modeled as

$$d\tilde{w}(\omega) = S_v^{\frac{1}{2}}(\omega)\, \Psi(j\omega)\, dw(\omega), \tag{19}$$

in which $\Psi(j\omega)$ is the $N \times m$-matrix mentioned in Subsection 3.1. It is a simple exercise to study that this $d\tilde{w}(\omega)$ has properties (15) and (16).

Substituting (19) into (14), we have the expression

$$v(t) = \int_{-\infty}^{\infty} S_v^{\frac{1}{2}}(\omega)\ \Psi(j\omega)\, e^{j\omega t}\, dw(\omega). \tag{20}$$

This is another model for generating the wind velocity vector whose spectral density matrix is given by $S_v(\omega)$.

At this stage it will be worthy to check statistical properties of $v(t)$-process generated by (20). Clearly, its mean is zero; while and the correlation is given as

$$\begin{aligned} R_v(\tau) &= \mathcal{E}\{v(t+\tau)v^*(t)\} \\ &= \mathcal{E}\left\{\left[\int_{-\infty}^{\infty} S_v^{\frac{1}{2}}(\omega_1)\ \Psi(j\omega_1)\, e^{j\omega_1(t+\tau)}\, dw(\omega_1)\right]\right. \\ &\qquad\qquad \left.\cdot \left[\int_{-\infty}^{\infty} S_v^{\frac{1}{2}}(\omega_2)\ \Psi(j\omega_2)\, e^{j\omega_2 t}\, dw(\omega_2)\right]^*\right\} \\ &= \int_{-\infty}^{\infty} S_v^{\frac{1}{2}}(\omega_1)\, e^{j\omega_1\tau}\left[\int_{-\infty}^{\infty} \mathcal{E}\{\ \Psi(j\omega_1)\mathcal{E}\{dw(\omega_1)\, dw^T(\omega_2)\}\right. \\ &\qquad\qquad \left. \cdot \Psi^*(j\omega_2)\}\, S_v^{\frac{1}{2}}(\omega_2)\, e^{j(\omega_1-\omega_2)t}\right] \\ &= \int_{-\infty}^{\infty} S_v^{\frac{1}{2}}(\omega_1)\, e^{j\omega_1\tau}\left[\mathcal{E}\{\ \Psi(j\omega_1)\mathcal{E}\{dw(\omega_1)\, dw^T(\omega_2)\}\, \Psi^*(j\omega_2)\}\, S_v^{\frac{1}{2}}(\omega_2)\right]_{\omega_1=\omega_2} \\ &= \int_{-\infty}^{\infty} S_v(\omega_1)\, e^{j\omega_1\tau}\, d\omega_1 \end{aligned} \tag{21}$$

which is just the Wiener-Khinchin formula for stationary process and proves that the spectral density of the $v(t)$-process is $S_v(\omega)$.

4. **Generation of Windward Pressure.** Let us proceed to obtain the model for generating the profiles of windward pressure. Let $p(t) = [p_1(t), \cdots, p_N(t)]^T$ be the N-vector consisting of the windward pressure $p_i(t)$ at ith story. As well known in the architecture community, the transfer nature from the wind velocity to its pressure is given in terms of the aerodynamic admittance. Based on this fact, let us assume the linear relation between wind velocity $v(t)$ and the pressure $p(t)$.

Here, let $h_{p_i v_k}(\tau)$ be the impulse response between $v_k(t)$ and $p_i(t)$ $(i, k = 1, 2, \cdots, N)$. Then, we have

$$p_i(t) = \sum_{k=1}^{N} \int_0^\infty h_{p_i v_k}(\tau)\, v_k(t-\tau)\, d\tau, \tag{22}$$

or in a vector form

$$p(t) = \int_0^\infty H_{pv}(\tau)\, v(t-\tau)\, d\tau, \tag{23}$$

where $H_{pv}(\tau) = [\, h_{p_i v_k}(\tau)\,]_{i,k=1,2,\cdots,N}$ is the impulse response matrix.

Given the frequency response matrix $G_{pv}(j\omega)$ of the wind velocity $v(t)$, the impulse response matrix $H_{pv}(\tau)$ can be calculated by

$$H_{pv}(\tau) = \frac{1}{2\pi} \int_{-\infty}^\infty G_{pv}(j\omega)\, e^{j\omega\tau}\, d\omega. \tag{24}$$

The aerodynamic admittance is given by [8]

$$\chi_i(j\omega) = \frac{1}{1 + \left(j\omega \dfrac{\sqrt{a_i}}{\pi \bar{v}_i} \right)^{4/3}} \qquad (i = 1, 2, \cdots, N), \tag{25}$$

so we assume the frequency transfer function from $v_k(t)$ to $p_i(t)$ as

$$[\, G_{pv}(j\omega)\,]_{ii} = A_i \chi_i(j\omega) \tag{26}$$

where $A_i = \rho\, C_{Di}\, a_i \bar{v}_i$ in which ρ is the air density, C_{Di} the windward drag coefficient at height z_i, a_i the tributary area for the ith story unit, and $\bar{v}_i$ is the mean along-wind velocity at height z_i. The profile of windward pressure $p(t)$ can be obtained by (23).

The power spectral density $S_p(\omega)$ of the $p(t)$-process is calculated as follows. Now, the correlation matrix of $p(t)$ is evaluated from (23) as

$$\begin{aligned} R_p(\tau) &= \mathcal{E}\{p(t+\tau)p^T(t)\} \\ &= \mathcal{E}\left\{ \left[\int_0^\infty H_{pv}(\sigma_1) v(t+\tau-\sigma_1)\, d\sigma_1 \right] \left[\int_0^\infty H_{pv}(\sigma_2) v(t-\sigma_2)\, d\sigma_2 \right]^T \right\} \\ &= \int_0^\infty \int_0^\infty H_{pv}(\sigma_1)\, \mathcal{E}\{v(t+\tau-\sigma_1) v^T(t-\sigma_2)\} H_{pv}^T(\sigma_2)\, d\sigma_2\, d\sigma_1 \\ &= \int_0^\infty \int_0^\infty H_{pv}(\sigma_1)\, R_v(\tau-\sigma_1+\sigma_2) H_{pv}^T(\sigma_2)\, d\sigma_2\, d\sigma_1. \end{aligned} \tag{27}$$

Fourier-transforming this, we have

$$
\begin{aligned}
S_p(\omega) &= \int_{-\infty}^{\infty} R_p(\tau)\, e^{-j\omega\tau} d\tau \\
&= \int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} H_{pv}(\sigma_1)\, e^{-j\omega\sigma_1} R_v(\tau-\sigma_1+\sigma_2)\, e^{-j\omega(\tau-\sigma_1+\sigma_2)} \\
&\qquad \cdot H_{pv}^T(\sigma_2)\, e^{j\omega\sigma_2}\, d\sigma_2\, d\sigma_1\, d\tau \\
&= \int_{0}^{\infty} H_{pv}(\sigma_1)\, e^{-j\omega\sigma_1} \left[\int_{-\infty}^{\infty} R_v(\tau-\sigma_1+\sigma_2)\, e^{-j\omega(\tau-\sigma_1+\sigma_2)} d\tau\right] \\
&\qquad \cdot \int_{0}^{\infty} H_{pv}^T(\sigma_2)\, e^{j\omega\sigma_2}\, d\sigma_2\, d\sigma_1 \\
&= \left[\int_{0}^{\infty} H_{pv}(\sigma_1)\, e^{-j\omega\sigma_1} d\sigma_1\right] S_v(\omega) \left[\int_{0}^{\infty} H_{pv}^T(\sigma_2)\, e^{j\omega\sigma_2} d\sigma_2\right] \\
&= G_{pv}(j\omega) S_v(\omega) G_{pv}^*(j\omega)
\end{aligned}
\tag{28}
$$

which gives the power spectral density of $p(t)$.

5. **Procedure of Generating Wind Velocity and Windward Pressure Profiles.** Consequently, the procedure for generating profiles of wind velocity $v(t)$ and windward pressure $p(t)$ is summarized as follows:

Step 1. *Preparation of power spectral density matrix* $S_v(\omega)$. First, according to (2), obtain the matrix $S_v(\omega)$.

Step 2. *Generation of white noise process* $\{\gamma(t)\}$, *or Wiener process* $\{w(\omega)\}$. Generate m white Gaussian noise processes $\{\gamma_i(t), i = 1, 2, \cdots, m\}$, or m standard Wiener processes $\{w_i(\omega), i = 1, 2, \cdots, m\}$ on $-\infty < \omega < \infty$.

Step 3. *Generation of wind velocity* $v(t)$. With an appropriately equipped function $\Psi(j\omega)$, compute $v(t)$ by (5), or by (20).

Step 4. *Generation of windward pressure* $p(t)$. Using $v(t)$ obtained in Step 3, compute $p(t)$ by (23) with (24) and (26).

When we use the model (20), the process $v(t)$ can be generated by a computer using its discretized version. Let $[-L, L]$ be a (sufficiently) large interval on the ω-axis, and partition it into n segments. Then, (20) can be approximated as

$$
v(t) = \sum_{\ell=0}^{n-1} S_v^{\frac{1}{2}}(\omega_\ell)\, \Psi(j\omega_\ell)\, e^{j\omega_\ell t}\, \delta w(\omega_\ell), \tag{29}
$$

where $\delta w(\omega_\ell)$ is the increment of $w(\omega)$- process at $\omega = \omega_\ell$. Using uniform random numbers $\{n_\ell, \ell = 0, 1, 2, \cdots, n-1\}$ ($n_\ell \sim N[0, 1]$) generated by a computer, the increment $\delta w(\omega_\ell)$ can be computed by $\delta w(\omega_\ell) = n_\ell \sqrt{\delta\omega_\ell}$ ($\delta\omega_\ell = \omega_{\ell+1} - \omega_\ell$) [12]. Therefore, by taking the real part of the right-hand side of (29), we obtain the computer version

$$
v(t) = \sum_{\ell=0}^{n-1} S_v^{\frac{1}{2}}(\omega_\ell)\, \mathrm{Re}\,\{\Psi(j\omega_\ell)\, e^{j\omega_\ell t}\}\, n_\ell \sqrt{\delta\omega_\ell}. \tag{30}
$$

By controlling the function $\Psi(j\omega)$ the random process $v(t)$ can be generated by computers.

It should be noted here that our procedure is rather simpler than that proposed in [6]. Our procedure for generating wind profiles does not introduce any differential or difference equation such as autoregressive models but requires only the function $\Psi(j\omega)$ of the form (11) or (12).

6. **Discussions.** (i) Since the method for generating the wind velocity $v(t)$ proposed in Subsection 3.1 is based on the linear system response with the seed (vector) white Gaussian input $\gamma(t)$, the windward pressure $p(t)$ can be generated more directly as follows.

Substituting (6) into (28), we have the relation

$$\begin{aligned} S_p(\omega) &= G_{pv}(j\omega)\, G_{v\gamma}(j\omega)\, W\, G^*_{v\gamma}(j\omega)\, G^*_{pv}(j\omega) \\ &= G_0(j\omega)\, W\, G^*_0(j\omega), \end{aligned} \tag{31}$$

where

$$G_0(j\omega) = G_{pv}(j\omega)\, G_{v\gamma}(j\omega). \tag{32}$$

This $G_0(j\omega)$ gives the direct transfer matrix from the input $\gamma(t)$ to the output $p(t)$. So, if we let $H_0(\cdot)$ be the $N \times m$-impulse response matrix corresponding to $G_0(j\omega)$, i.e.,

$$p(t) = \int_0^\infty H_0(\tau)\, \gamma(t-\tau)\, d\tau. \tag{33}$$

Since

$$G_0(j\omega) = \int_{-\infty}^\infty H_0(t)\, e^{-j\omega t}\, dt \qquad (H_0(t) = 0 \text{ for } \ t < 0), \tag{34}$$

we get by inverting this

$$H_0(t) = \frac{1}{2\pi} \int_{-\infty}^\infty G_0(j\omega)\, e^{j\omega t}\, d\omega. \tag{35}$$

(ii) According to Yang [13], the expression (20) may be replaced by

$$\tilde{v}(t) = \int_{-\infty}^\infty S_v^{\frac{1}{2}}(\omega)\, e^{j\{\omega t - \theta(\omega)\}}\, \sqrt{d\omega} \tag{36}$$

in which $\Psi(j\omega)$ is set similarly to (11) and $\theta(\omega)$ is a uniform random number over $[0, 2\pi)$. In this expression, the integration by the term $\sqrt{d\omega}$ may seem quite strange from the mathematical point of view. However, the derivation of (36) can be shown heuristically as follows.

Assume the Fourier integral,

$$\tilde{v}(t) = \int_{-\infty}^\infty A(\omega)\, e^{j\{\omega t - \theta(\omega)\}}\, d\omega, \tag{37}$$

where $A(\omega)$ is an N-vector nonrandom function to be determined. The mean of this process is zero and the correlation matrix $R_{\tilde{v}}(\tau)$ is

$$R_{\tilde{v}}(\tau) = \int_{-\infty}^\infty A(\omega) A^T(\omega)\, e^{j\omega\tau}\, (d\omega)^2. \tag{38}$$

(See Appendix for the statistical properties of $\tilde{v}(t)$- process). From the Wiener-Khinchin formula we have

$$R_{\tilde{v}}(\tau) = \int_{-\infty}^\infty S_v(\omega)\, e^{j\omega\tau}\, d\omega. \tag{39}$$

Comparing (38) with (39), we see that

$$\int_{-\infty}^\infty A(\omega) A^T(\omega)\, e^{j\omega\tau}\, (d\omega)^2 = \int_{-\infty}^\infty S_v(\omega)\, e^{j\omega\tau}\, d\omega$$

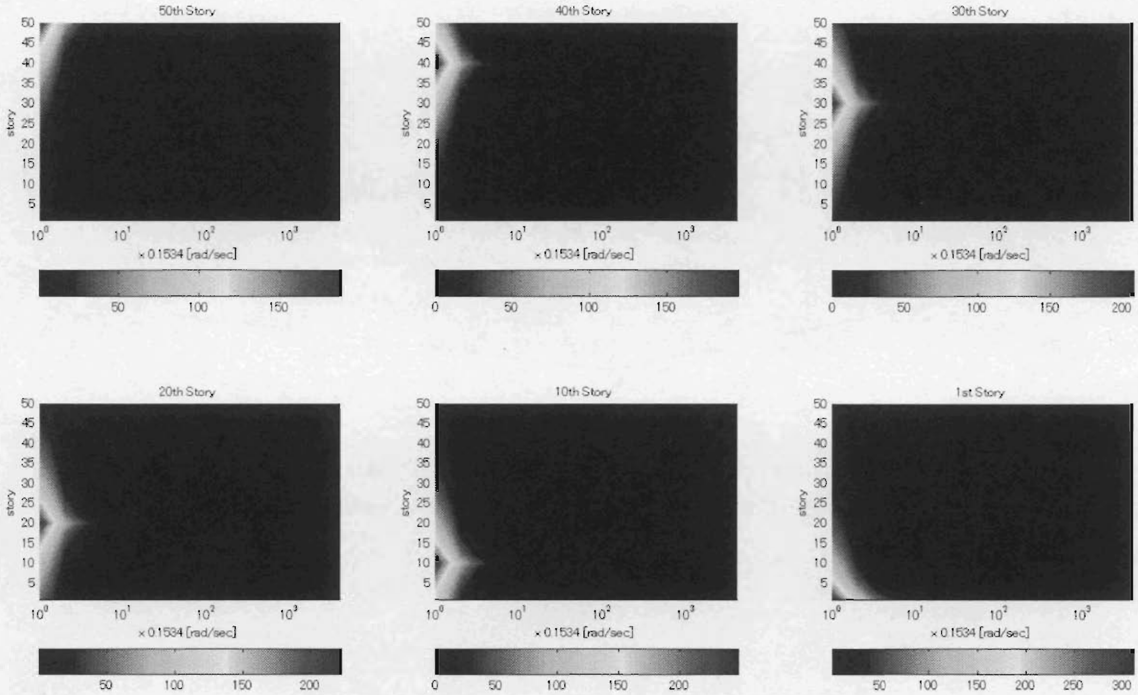

FIGURE 1. Given cross spectral densities $[S_v(\omega)]_{ik}$ modeled by (2). $[S_v(\omega)]_{ik}$ are listed for $i = 1, 10, 20, 30, 40$ and 50 from right to left in the lower row and to the upper row.

from which there follows

$$A(\omega)A^T(\omega)\,(d\omega)^2 = S_v(\omega)\,d\omega$$

or

$$A(\omega)\,d\omega = S_v^{\frac{1}{2}}(\omega)\,\sqrt{d\omega}. \tag{40}$$

Substituting (40) into (37), we obtain the expression (36).

Recalling that the order of the increment of Wiener process $dw(\omega)$ is $\sqrt{d\omega}$ and comparing (36) with (20), we may understand that the conventional expression (36) has the meaning in this sense.

7. **Simulation Study.** Assuming a fifty-story uniform building ($N = 50$) with each story height 4 m, simulations on the wind velocity were conducted.

As for the given power spectral density the von Kármán model (1) is employed and the parameters were set as: $\sigma_{v_i}^2 = 1$, $b_i = -0.05$, $V_g = 20$ m/s, $h_g = 10$ m, $\alpha = 0.27$, $c = 8$, $V_r = 20$; and the function $\psi_i(j\omega)$ given by (13) was assumed with $c_0 = 8$ for all i. For the model of the aerodynamic admittance the parameters were set: $\rho = 0.125 \times 9.8$, $C_{Di} = 0.8$, $a_i = 36 \times 4$ (= width of structure × strory heigh) m^2, and $\bar{v}_i = V_g\,(h_i/h_g)^{\alpha}$, respectively.

Figure 1 illustrates the given cross spectral densities modeled by (2) with density due to von Kármán (1). Figures 2 and 3 are the simulation results of the wind profiles $\{v_i(t)\}$ at stories $i = 1, 10, 20, 30, 40$ and 50 (top-story) by the methods proposed in Subsections 3.1 and 3.2, respectively, by letting $m = N$ (= 50). For computation, the sampling time interval was set as $\delta t = 0.01$ s. As for the numerical computation of (30), the interval $[-L, L] = [-314, 314]$ was $n = 4096$ equi-partitioned to obtain $\delta\omega_\ell = 0.153$.

From both figures the profiles of wind velocity seem well simulated.

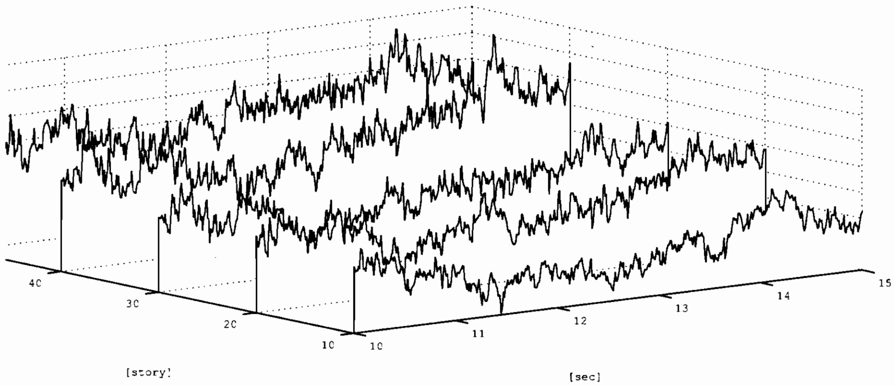

FIGURE 2. Generated wind velocities $v_i(t)$ at each story ($i = 1, 10, 20, 30, 40, 50$) by the method proposed in Subsection 3.1.

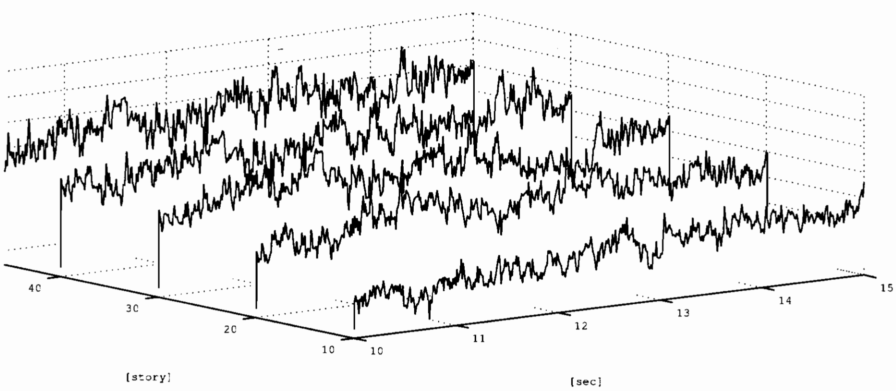

FIGURE 3. Generated wind velocities $v_i(t)$ by the method proposed in Subsection 3.2.

8. **Conclusion.** Using the von Kármán power spectral density function incorporated with Davenport coherence function, two methods for generating profiles of wind velocity along the building height have been proposed. The one is based on the linear dynamic system model whose inputs are white Gaussian noise processes, and the other employs the spectral representation model in which Wiener process on the frequency domain is used. The given power and cross spectra are imbedded in the frequency transfer matrix, or in the spectral representation. Once the power and cross spectra of the wind velocity have been given, the generation of wind velocity profile can be readily realized by any one of the methods proposed in this paper. In this sense the proposed two methods will have potential ability for the analysis and/or design of high-rise building subjected to random wind loads.

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Appendix. *Statistical properties of $\tilde{v}(t)$-process.*

The mean is

$$\mathcal{E}\{\tilde{v}(t)\} = \int_{-\infty}^{\infty} A(\omega)\, e^{j\omega t}\, \mathcal{E}\{e^{-j\theta(\omega)}\}\, d\omega$$

in which $\mathcal{E}\{e^{-j\theta(\omega)}\} = 0$ since

$$\begin{aligned}\mathcal{E}\{e^{-j\theta(\omega)}\} &= \int_0^{2\pi} [\cos\theta - j\sin\theta]\, p(\theta)\, d\theta \quad \left(p(\theta) = \frac{1}{2\pi}\right)\\ &= \frac{1}{2\pi}\int_0^{2\pi} \cos\theta\, d\theta - j\frac{1}{2\pi}\int_0^{2\pi} \sin\theta\, d\theta = 0.\end{aligned}$$

The correlation is

$$\begin{aligned}R_{\tilde{v}}(\tau) &= \mathcal{E}\{\tilde{v}(t+\tau)\tilde{v}^*(t)\}\\ &= \mathcal{E}\left\{\left[\int_{-\infty}^{\infty} A(\omega_1)\, e^{j\{\omega_1(t+\tau)-\theta(\omega_1)\}}\, d\omega_1\right]\left[\int_{-\infty}^{\infty} A(\omega_2)\, e^{j\{\omega_2 t-\theta(\omega_2)\}}\, d\omega_2\right]^*\right\}\\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\omega_1)A^T(\omega_2)\, e^{j\{(\omega_1-\omega_2)t+\omega_1\tau\}}\, \mathcal{E}\{e^{-j\{\theta(\omega_1)-\theta(\omega_2)\}}\}\, d\omega_2\, d\omega_1\end{aligned}$$

in which

$$\mathcal{E}\{e^{-j\{\theta(\omega_1)-\theta(\omega_2)\}}\} = \begin{cases} 1 & (\omega_1 = \omega_2)\\ 0 & (\omega_1 \neq \omega_2).\end{cases}$$

So,

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A(\omega_1)A^T(\omega_2)\, e^{j\{(\omega_1-\omega_2)t+\omega_1\tau\}} \begin{Bmatrix} 1 & (\omega_1 = \omega_2) \\ 0 & (\omega_1 \neq \omega_2) \end{Bmatrix} d\omega_2\, d\omega_1$$

$$= \int_{-\infty}^{\infty} A(\omega_1)\, e^{j\omega_1\tau} \left[\int_{-\infty}^{\infty} A^T(\omega_2)\, e^{j(\omega_1-\omega_2)t}\, d\omega_2\right]_{\omega_2=\omega_1} d\omega_1$$

$$= \int_{-\infty}^{\infty} A(\omega_1)A^T(\omega_1)\, e^{j\omega_1\tau}(d\omega_1)^2$$

which is just (38).